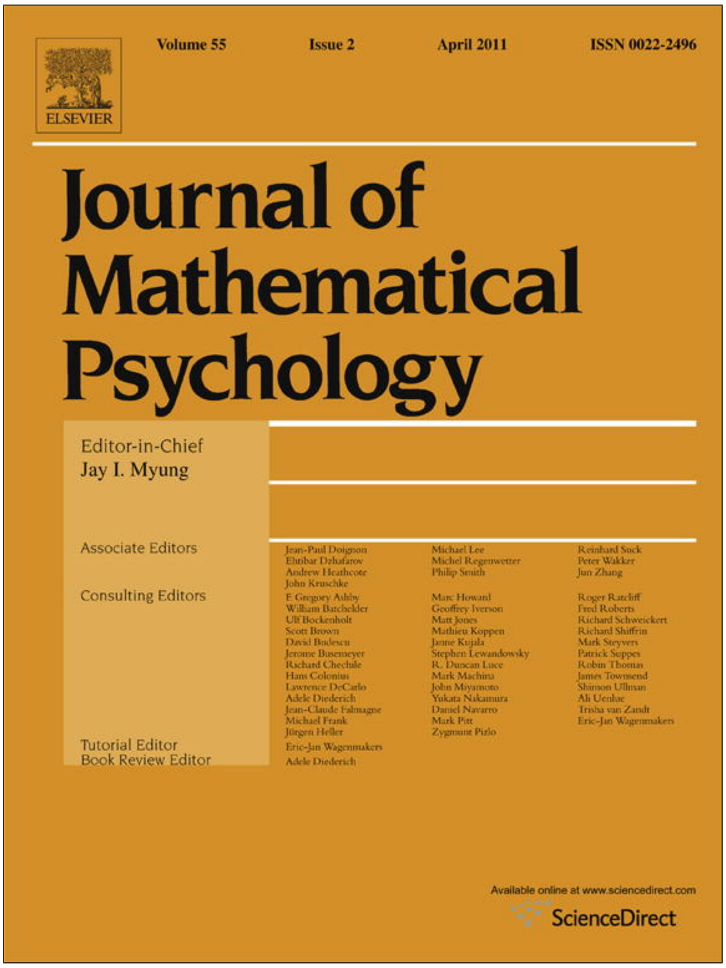
Volume 55
Issue 2
April 2011
ISSN 0022-2496
ELSEVIER
Journal of Mathematical Psychology
Editor-in-Chief
Jay I. Myung
Associate Editors
Consulting Editors
Tutorial Editor
Book Review Editor
Available online at www.sciencedirect.com
ScienceDirect

Journal of Mathematical Psychology 55 (2011) 176–190

Contents lists available at ScienceDirect

Journal of Mathematical Psychology

journal homepage: www.elsevier.com/locate/jmp

Nice guys finish fast and bad guys finish last: Facilitatory vs. inhibitory interaction in parallel systems

Ami Eidels [a,*], Joseph W. Houpt [b], Nicholas Altieri [b], Lei Pei [b], James T. Townsend [b]

[a] *University of Newcastle, Australia*

[b] *Indiana University, Bloomington, United States*

ARTICLE INFO

Article history:
Received 25 December 2009
Received in revised form
20 November 2010
Available online 21 December 2010

Keywords:
Parallel
Interactive
Processing
Survivor interaction contrast
Capacity
Response time

ABSTRACT

Systems Factorial Technology is a powerful framework for investigating the fundamental properties of human information processing such as architecture (i.e., serial or parallel processing) and capacity (how processing efficiency is affected by increased workload). The Survivor Interaction Contrast (SIC) and the Capacity Coefficient are effective measures in determining these underlying properties, based on response-time data. Each of the different architectures, under the assumption of independent processing, predicts a specific form of the SIC along with some range of capacity. In this study, we explored SIC predictions of discrete-state (Markov process) and continuous-state (Linear Dynamic) models that allow for certain types of cross-channel interaction. The interaction can be facilitatory or inhibitory: one channel can either facilitate, or slow down processing in its counterpart. Despite the relative generality of these models, the combination of the architecture oriented plus the capacity oriented analyses provide for precise identification of the underlying system.

The issue of how we process multiple signals or multiple attributes of a given object is of considerable interest to psychologists. Different signals can be processed simultaneously (i.e., in a parallel manner) or sequentially (i.e. in a serial manner). Additionally, the signals can be processed in independent channels, or alternatively, the channels can somehow communicate with each other in such a way that one channel facilitates or inhibits processing in the other channel. In this paper we explore response-time (RT) predictions of parallel models that allow some degree of cross-channel interactions.

The following example will serve us throughout this report: suppose that two sources of information, say, an auditory and a visual signal, are processed in parallel channels 1 and 2 respectively. The channels can operate independently from one another, as shown in Fig. 1A. That is, the activation in channel 1 does not affect the activation level in channel 2, and vice versa. Conversely, the channels may interact, as in Fig. 1C. The interaction can be positive where each channel facilitates the processing of its counterpart causing an overall reduction in the time it takes to finish the processing of the incoming information. Hence, *nice guys finish fast*. Alternatively, the channels may inhibit each other's activity causing a slowdown in performance and *bad guys finish last*.

In the absence of direct access to the underlying mental processes, researchers have traditionally adopted behavioral measures such as mean RTs to assess how different, most often simultaneously presented signals are processed (e.g. Donders, 1868; Sternberg, 1969). Investigators have generally been concerned with broad information processing issues such as whether multiple sources of information are processed in serial or in parallel. However, these techniques typically assume independent processing in the respective channels and little research has been carried out to investigate the effects of dependencies between processing channels.

One shortcoming of methodologies traditionally used to assess parallel versus serial processing is that mean RTs alone often cannot differentiate between competing models. Serial and parallel systems may mimic each other by exhibiting the same pattern of observed response times (e.g., Snodgrass & Townsend, 1980; Townsend, 1972, 1990a). For example, Snodgrass and Townsend showed how parallel models with limited capacity can easily mimic broad classes of serial models. A related issue is a possible trade-off between processing capacity and architecture, in which RT measures are consistent with parallel processing while capacity is in some sense 'limited' and consistent with serial processing (cf. Townsend & Ashby, 1983). *Workload capacity*, or simply *capacity*, refers to the system's performance when the load is varied.[1] If the processing rate on one channel remains invariant

* Corresponding address: School of Psychology, The University of Newcastle, Callaghan, NSW 2308, Australia.
E-mail address: ami.eidels@newcastle.edu.au (A. Eidels).

0022-2496/$ – see front matter © 2010 Elsevier Inc. All rights reserved.
doi:10.1016/j.jmp.2010.11.003

[1] The term *capacity* is used elsewhere for different purposes. We confine our definition of the term "capacity" to the relative change in speed by which the

when another signal is added, then the capacity of the system is *unlimited*. Alternatively, if increasing the work load by presenting an additional signal slows down processing in a given channel then capacity is *limited*.

To overcome the problem of model mimicking, Townsend and colleagues (e.g. Schweickert & Townsend, 1989; Townsend, 1984) built on the concepts of *selective influence* and *mean interaction contrast*. These concepts were first brought to the attention of experimental psychologists by Sternberg (1969). For an experimental manipulation to 'selectively influence' a particular process, the manipulation must affect a designated hypothetical process and no other process. For example, a sound intensity manipulation is said to selectively influence the auditory channel if it affects processing of the auditory signal but has no effect on processing of the visual signal (Schweickert & Townsend, 1989; Townsend & Schweickert, 1989; see also Townsend, 1990b). The mean interaction contrast computes a double difference of the empirical mean RTs across, for example, the 2 × 2 manipulations of an orthogonal 2 × 2 factorial design. Dzhafarov and colleagues (Dzhafarov, 2003; Kujala & Dzhafarov, 2008) have provided advances in the theory and methodology associated with the vital assumption of selective influence. When selective influence is abrogated, it has been demonstrated that the standard predictions of parallel vs. serial models on the mean interaction contrast statistic fall apart (Townsend & Thomas, 1994).

One challenge accompanying mean interaction contrast analyses was that certain architectures combined with certain stopping rules made identical predictions on that statistic. Subsequently to the theoretical results on mean interaction contrast, more robust statistical measures that utilize entire RT distributions rather than mean RTs were devised (Townsend & Nozawa, 1995; Townsend & Wenger, 2004a). The outcome was a contrast statistic, the *survivor interaction contrast* (SIC) which met the goal of using the entire RT distribution to assess architectures and stopping rules. This statistic permits a considerably finer-grained analysis of the latter characteristics.

Based on this new statistic, plus that of a measure of workload capacity, Townsend and Nozawa (1995) developed a mathematical theory in combination with a related methodology dubbed *Systems Factorial Technology*. In consequence, they proposed an associated experimental design, called the *double factorial design*, to distinguish between serial and parallel processing architectures and within the latter category, independent-parallel from coactive-parallel models.[2] The operative stopping rule is also assayed.

The SIC statistic (also called an *index*) will be defined formally in the next section, but briefly, parallel and serial models predict unique functional forms for the SIC. For example, suppose that a human observer is asked to respond affirmatively if an auditory signal *and* visual signal both appear. To respond correctly, the observer must exhaustively process both modalities. Under this regime, if the two signals are processed in parallel, then the predicted survivor contrast is negative, as depicted in Fig. 2B. If the signals are processed serially, then the predicted SIC has a distinctive S-shaped curve that begins at zero and then becomes negative, crosses the abscissa, and then becomes positive before returning to zero. The SIC signatures for serial models are presented elsewhere (Townsend & Nozawa, 1995). In this paper we examined the effects that different levels of cross-channel interaction have on the SIC signature of parallel models.

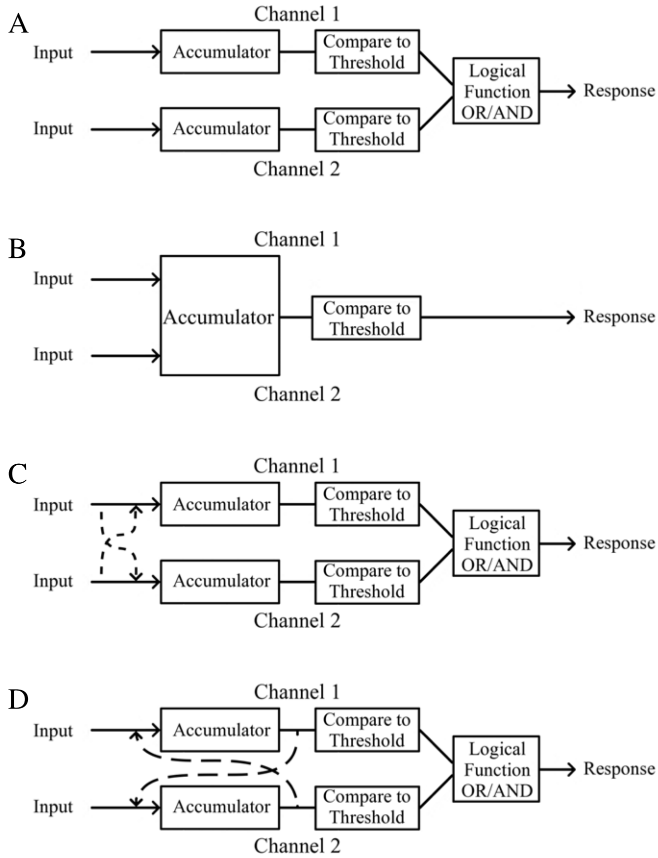

Fig. 1. Schematics of four types of parallel processing models: independent parallel channels (panel A), parallel coactive model (panel B), parallel channels with pre-accumulator interaction (panel C), and parallel channels with post-accumulator interaction (panel D).

If the channels in a parallel system interact with each other then the experimental manipulation targeted on one channel will have an effect on the other, violating the assumption of selective influence. Unlike 'pure' parallel or serial models, the channels are no longer independent; activation from one channel, such as the auditory channel, may be sent to the other channel and vice versa. The outcome of this cross-channel communication may be facilitatory or inhibitory depending on the nature of the interaction. In the current study we examined several classes of formal and computational parallel-interactive models, and explored their predictions with respect to the SIC and workload capacity, beyond the cases where selective influence holds.

The SIC test is traditionally employed within the context of a factorial design. We begin by outlining the paradigm often referred to as "the double factorial design". We then explain the basic methodology for calculating the SIC and discuss the predictions for parallel independent models. Next, we describe two types of models, discrete state and continuous state, that are used to explore early cross-channel interactions (pre-accumulator) and late interactions (post-accumulator).[3] We then report simulation results of these models in terms of the SIC and workload capacity patterns they predict. Finally, we discuss the similarities and differences in the predicted SICs due to changes in the locus in which interactions occur.

cognitive system processes information as we increase the workload by increasing the amount of to-be-processed information.

[2] In a coactive model, activation from multiple channels is summed and compared to a single threshold prior to the decision. In the case of the Poisson coactive model, for example, counts from two or more channels can accumulate in a common buffer, in which the overall amount of counts is subsequently compared to the decision criterion.

[3] Cross-channel interaction may be early on in the process, representing perhaps a dependence of the activation in one channel on the input from the other. Or else, the interaction may occur at a later stage, for example if the activation in a channel depends on the activation in the other.

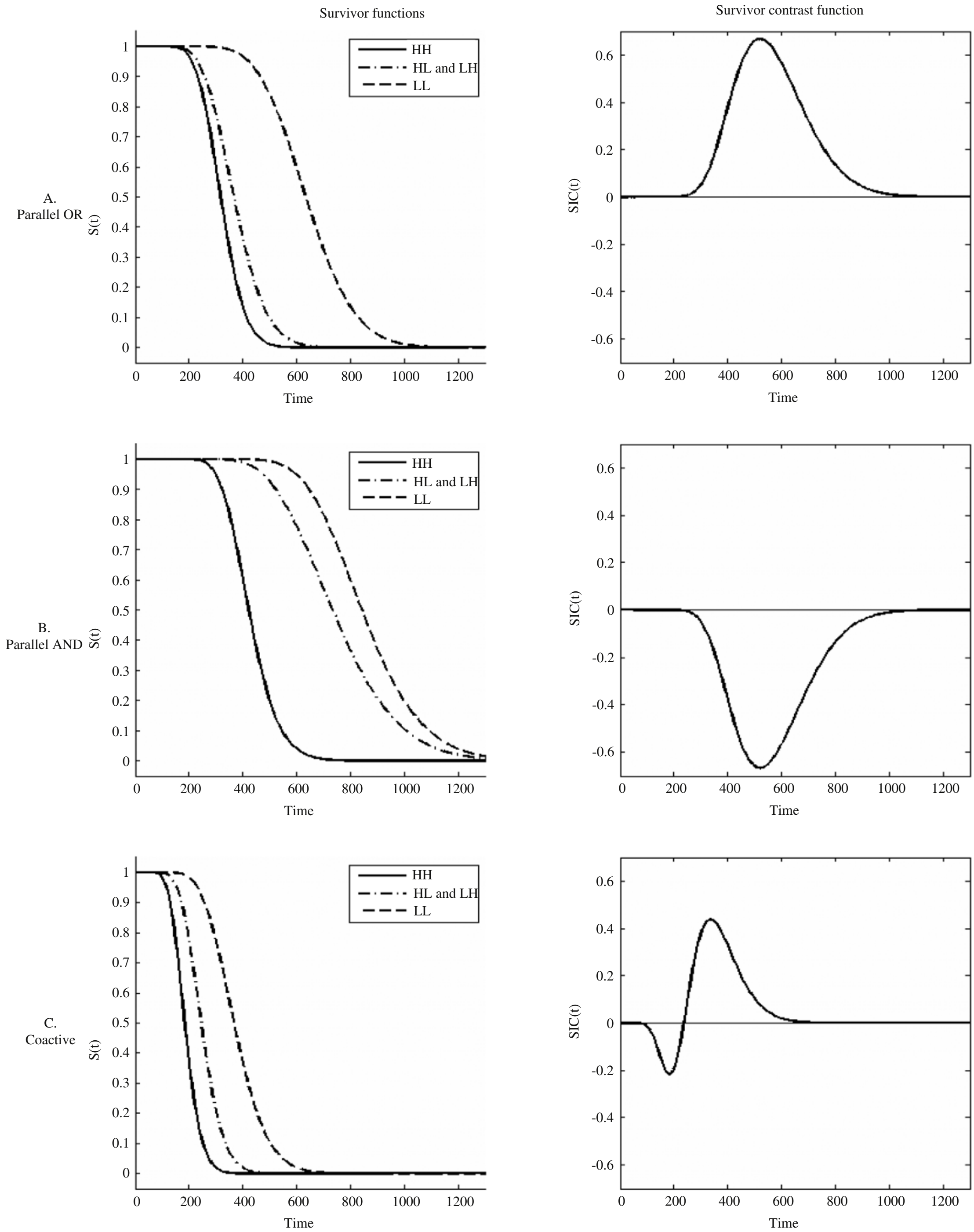

Fig. 2. Survivor functions (left column) and SIC predictions (right column) for different processing models: parallel first-terminating (panel A), parallel exhaustive (panel B), and coactive (panel C). To calculate the SIC, one first estimates the survivor functions for each of the four factorial conditions (HH, HL, LH, and LL), and then calculates the double difference: $\mathrm{SIC}(t) = [S_{\mathrm{LL}}(t) - S_{\mathrm{LH}}(t)] - [S_{\mathrm{HL}}(t) - S_{\mathrm{HH}}(t)]$. The subscripts refer to the salience of the signals, where H is high and L is low. So, for example, HH means that both signals are highly salient.

1. The double factorial design

The double factorial design combines two levels of manipulation. The first manipulation is concerned with the presence versus absence of target items. For instance, in a target detection task with auditory and visual targets, four types of trials exist: double target trials, in which an auditory signal and a visual signal are presented at the same time, visual target alone, auditory target alone, and finally target absent trials. This manipulation of presence versus absence is used to create double versus single target conditions, which are necessary for the calculations of our capacity measure, as we shall see in later sections.[4] A second manipulation of salience performed on the subset of double target trials yields four subtypes of trials: HH trials, where both the visual and the auditory target appear in their highly salient form (for example, a loud beep sound and a bright dot of light), HL and LH trials, where one target

[4] Target-absent trials are not used for the calculation of our capacity measure, yet are essential in this paradigm. Without them, all displays contain at least one target item, so participants would be able to correctly respond "yes, target-present" without actually processing the stimuli.

is highly salient whereas the salience level of the other target is low (e.g., loud sound and a dim dot, or a bright dot and weak sound), and LL trials where both targets have low salience.

The survivor function for each of the factorial conditions (HH, LH, HL, and LL) can then be estimated from response times to yield the SIC. The survivor function is the complement of the cumulative distribution function (CDF), such that $S(t) = 1 - F(t)$. While the cumulative distribution function, $F(t)$, tells us the probability that processing of a given stimulus is finished before or at time t, the survivor function marks the probability that processing has not yet terminated. The SIC is computed by taking a double difference of survivor functions from the different factorial conditions created by the high vs. low salience manipulation, $\mathrm{SIC}(t) = [S_{\mathrm{LL}}(t) - S_{\mathrm{LH}}(t)] - [S_{\mathrm{HL}}(t) - S_{\mathrm{HH}}(t)]$.

The SIC predictions for two independent parallel models are presented in panels A and B of Fig. 2 (for formal proofs and predictions for serial models, we refer the reader to Townsend & Nozawa, 1995). Townsend and Nozawa also derived predictions for a special case of parallel processing, referred to as coactive processing, in which information from two channels converges to satisfy a single criterion. A schematic of such a model is presented in Fig. 1B, and the SIC prediction is plotted in Fig. 2C. Under some assumptions, which we discuss later, the coactive model is in fact a special case of an interactive-facilitatory model.

We systematically varied the degree of cross-channel interaction within several classes of simulated models, and tested how it affects the form of the SIC. Varying the level of interaction makes parallel models flexible in terms of their predictions. In particular, it allows the model to mimic a range of architectures from independent-parallel (when the level of interaction is negligible or effectively null) to coactive. Consequently, parallel interactive models can predict a range of SIC signatures. Nonetheless, we found that despite the inherent flexibility of interactive-parallel models, their SIC functions do in fact span a finite range, thus allowing the falsification of certain classes of models based on observed data. For example, a facilitatory AND model (a system with two parallel channels which facilitate each other and stops as soon as the slower of the two finishes processing) can produce a range of SIC functions from completely negative to mostly positive. An entirely positive SIC, often observed in some of our studies (e.g. Eidels & Townsend, 2009; Eidels, Townsend, & Algom, 2010; Townsend & Nozawa, 1995), would allow one to reject this broad class of parallel models.

We explored, in this paper, both discrete-state and continuous-state models of parallel processing. For the former we used Markov-process matrix methods whereas for the latter we used Monte-Carlo simulations. We focused on two varieties of interaction, one at the input stage (pre-accumulator) and one during the accumulation stage. For each model, we assumed that processing of two or more sources of information is carried out simultaneously in parallel channels. We allowed either first-termination (i.e., terminate processing when either channel 1 or 2 finishes; OR rule) or exhaustive processing (i.e., terminate processing when both 1 and 2 channels finish; AND rule). Furthermore, in all models, we manipulated the level of excitatory and inhibitory cross-channel interactions. However, the exact manner by which one channel affects the other differed across the two varieties.

Next, we present the models in greater detail and explain how the cross-channel interaction is realized in the discrete and continuous classes of models. The interaction can be facilitatory, with one channel 'helping' the other, or inhibitory, where one channel slows down the processing of its counterpart. Therefore, for each class of models there exist four cases of interest: facilitatory interaction associated with an OR rule, facilitatory with an AND rule, inhibitory OR, and inhibitory AND. After describing the models we present the simulation results showing the SIC functions for different levels of interactions for each of these four cases.

2. 'Early' and 'late' cross-channel interactions

Each channel can interact with its counterpart in different loci. In Fig. 1C and D we illustrate two possible loci of interaction, which we have explored in detail. In Fig. 1C, 'early' interaction, the interaction occurs before the accumulator in both channels. We refer to these models as "*pre-accumulator interaction*" models. This type of interaction is a model for dependent inputs. In facilitatory models, higher input in one channel leads to more activation feeding into the accumulator of the other channel. In inhibitory models the higher input in one channel leads to lower input to the accumulator of the other channel.

In Fig. 1D, 'late' interaction, accumulated activation on one channel is added to (in the case of facilitation) or subtracted from (in the case of inhibition) the input of the other channel. In this type of model, it is the total activation, not just the input level of one channel that affects the other. We refer to these models as "*post-accumulator interaction*" models. Naturally, in facilitatory models higher total activation on one channel leads to higher input level in the other channel's accumulator, whereas in inhibitory models higher total activation leads to lower input.

3. Discrete- and continuous-state models

The pre- and post-accumulator types of interaction were realized in this study within two types of models: a discrete-state model, based on a Markov process, and a continuous state model, which is based on a stochastic linear dynamic system.

Discrete state models

We modeled discrete-state parallel-interactive processes with two parallel counting processes or channels. The input to each channel was a stream of counts that arrived randomly, but independently, at a constant rate until the channel was finished. The rate was determined by the salience level of an assumed stimulus processed by that channel (salient stimulus = high rate, faint stimulus = low rate). The specific parameter values used for this paper are reported in Appendix A. Each channel in the model accumulates counts until a prescribed criterion is reached. Channels could facilitate or inhibit each other by sharing positive or negative counts, respectively. For models of pre-accumulator interaction, only the most recent count could be shared. For models of post-accumulator interaction, any amount of the previously accumulated counts could be shared. In the AND case ("detect signal 1 and 2"), overall processing in the system ceased only when *both* channels reach their respective criterion. In the OR case, overall processing stopped once *either* channel 1 or 2 reaches its criterion. The following examples illustrate the process of counting with facilitatory versus inhibitory channel interaction. In the examples, we present the activation in the model as if it changes over discrete time steps. However, the models presented in this paper are in fact continuous-time processes, so 'steps' are only used here for the purpose of explication.

Consider first a facilitatory model, where the probability of cross-channel interaction is 1 in both directions—from channel 1 to 2, and from channel 2 to 1. This means that activation is fully shared between channels, but the exact manner differs across pre- and post-accumulation models. Both model varieties start with [0, 0]. Suppose that on the first step, a count occurs on one channel. In the pre-accumulator models with probability 1 of sharing, an incoming count on a given channel is also added to the other channel, setting the state of the system to [1, 1]. On the second step, a count occurs on the second channel but not on the first. Nonetheless, due to the interaction, the same count is also sent from the second to the first channel, and the updated state would be [2, 2]. Notice that in this extreme case the channels are perfectly correlated and will terminate processing at the same time (as long as their criterion values are identical). In the post-accumulator models, all accumulated counts are shared. If the state of the model is [2, 2], then all counts are shared from both channels to the other, increasing the state to [4, 4].

Alternatively, consider an inhibitory model where the probability of channels' interaction is again symmetric and equal to 1. Suppose that the model state is [2, 2] and a count is added to channel 1. With cross-channel inhibition, activation added to one channel is subtracted from the other in one of two ways, depending on the locus on interaction: in the pre-accumulator models the added count to channel 1 is simultaneously subtracted from channel 2, so the new state would be [3, 1]. In the post-accumulator model, in contrast, a count would be subtracted from channel 2 due to sharing from channel 1 at a rate proportional to $2p$ (since there are two counts in channel 1) and likewise for decreases in channel 1 due to sharing from channel 2. By assumption, a channel cannot have fewer than zero counts. For instance, if the model starts at [0, 0] and a count is added to channel 1, a count would not be subtracted from channel 2 even if the probability of interaction is $p = 1$. In that case, the updated state of the model becomes [1, 0].

A formal description of the discrete-state models is provided in Appendix A. We investigated the RT predictions, and in particular the SIC predictions of these models by carrying out simulations in some cases and numerical approximations to the CDFs in other cases. We tested both facilitatory and inhibitory models with varying levels of cross-channel interaction starting with completely independent channels, where the probability of interaction was null, $p = 0$, all the way through $p = 1$. In Appendix A we present the general model, but for brevity report in the main text results in which the sharing between channels is symmetric and the criteria are equal. Results from a wider range of parameters values are summarized in Appendix C, and in general were qualitatively similar to the canonical SIC forms which we report below.

Continuous state models

We modeled continuous-state parallel-interactive processes with linear dynamic systems. Similar to the discrete-state models, we specified a state space describing the accumulation of perceptual or cognitive activation in a channel at each point in time. The process of accumulation began when input entered the system from the environment or from another internal system. Again the salience level determined the magnitude of the input. To make the process stochastic we added independent white noise processes to the input. Pre-accumulator interactions were modeled by adding a multiple of the input of each channel to the other. Post-accumulator interaction was modeled by adding a multiple of the total activation of each channel to the other. The level of interaction was determined by the magnitude of the multiplier in either case. In facilitatory models the multiplier was positive, while in inhibitory models the multiplier was negative.

We simulated the models with varying levels of cross-channel interaction starting with completely independent channels and gradually increasing the extent of the interaction. To obtain the necessary estimate of the CDF in each condition, we simulated a series of trials with the model to get a sample of predicted RTs. From those estimated CDFs we computed and plotted the SIC. For simplicity, the interaction parameters were set to be equal across channels. For a formal explication of the continuous-state models see Appendix B.

4. Results and discussion

Simulation results for the models presented above are summarized in Fig. 3. The qualitative SIC predictions of the discrete-state and continuous-state models were the same. To avoid redundancy, we only included figures of the former. The SIC patterns predicted by pre- and post-accumulator models were often the same but differed on some aspects. Therefore we included figures of both, and compare their results shortly. These figures are based on our models' simulations, not on analytic proofs.

The SIC functions for four types of pre-accumulator model (facilitatory AND, facilitatory OR, inhibitory AND, inhibitory OR) are presented in the first column of Fig. 3. The corresponding SIC functions for the post-accumulator models are shown in the second column of Fig. 3. The solid black line in each panel corresponds to the SIC function of the parallel independent model. A lighter shade represents more interaction, with the lightest line representing the SIC function with the highest level of interaction. While the Markov process models have a clear maximum level of interaction ($p = 1$), the linear dynamic models are only bounded by the constraint on facilitation that the system remains stable and the constraint on inhibition that the system should complete processing in a finite time. For the parameters used in the simulation of the post-accumulator linear-dynamic models, this corresponded to cross-channel interaction values of $a_{12} = a_{21} = \pm 4.8$, where a_{12} determines the amount of cross channel information received by channel 1 from channel 2, and vice versa for a_{21} (see Appendix B for details).

A cursory comparison between the first and second columns of Fig. 3 reveals that the patterns of results predicted by pre- and post-accumulator model are qualitatively quite similar. Next, we survey the results of each class in more detail and point out discrepancies, when they exist. The order of discussion coarsely follows the difficulty for interpretation, from easy to more difficult, and not necessarily the order of presentation in Fig. 3.

Pre-accumulator models

For both facilitatory models (AND, OR; top two rows of Fig. 3), increasing the probability of interaction resulted in faster completion times. The corresponding curves shifted farther to the left as the level of facilitation increases (as the shade lightens). For the inhibitory model (bottom panels), increased interaction resulted in slower processing, and the corresponding SIC functions shifted to the right.

Fig. 3A shows the SIC functions for a facilitatory exhaustive (AND) model where two parallel channels facilitated each other and stopped as soon as both channels finished processing. For the independent parallel-exhaustive models (i.e., $p = 0$), the SIC function was entirely negative, like Fig. 2A, and commensurate with Townsend and Nozawa's (1995) Proposition 2. As the probability of cross-channel interaction increased, the early part of the survivor contrast function (i.e., for small t) remained negative, but the later part became more and more positive until, for p close to or equal to 1, the size of the positive area exceeded that of the early negative area. It is important to note that the facilitatory exhaustive model failed to produce a completely positive SIC function regardless of the amount of interaction. In fact, for the highest level of interaction the curve took the form of the SIC function predicted by a coactive model presented in Fig. 2C (see Townsend & Nozawa's Proposition 5). This result is predictable because perfectly correlated channels (cross-channel interactions of $p = 1$ in the discrete-state model) mean that all activation from one channel is sent to the other channel and vice versa. Hence, termination of processing on each channel occurred when the sum of counts from the two channels exceeds the criterion value, exactly as in a coactive (channel-summation) model.

Fig. 3B shows the SIC function for a facilitatory first terminating (OR) model. For $p = 0$ (i.e., no cross channel interaction) the SIC remained entirely positive for all t, as predicted by an independent parallel first-terminating model (Fig. 2A; see also Townsend & Nozawa, 1995, Proposition 1). As interaction increased, the early part of the function turned negative, but the total negative area was smaller than the positive area for all levels of interaction. At the

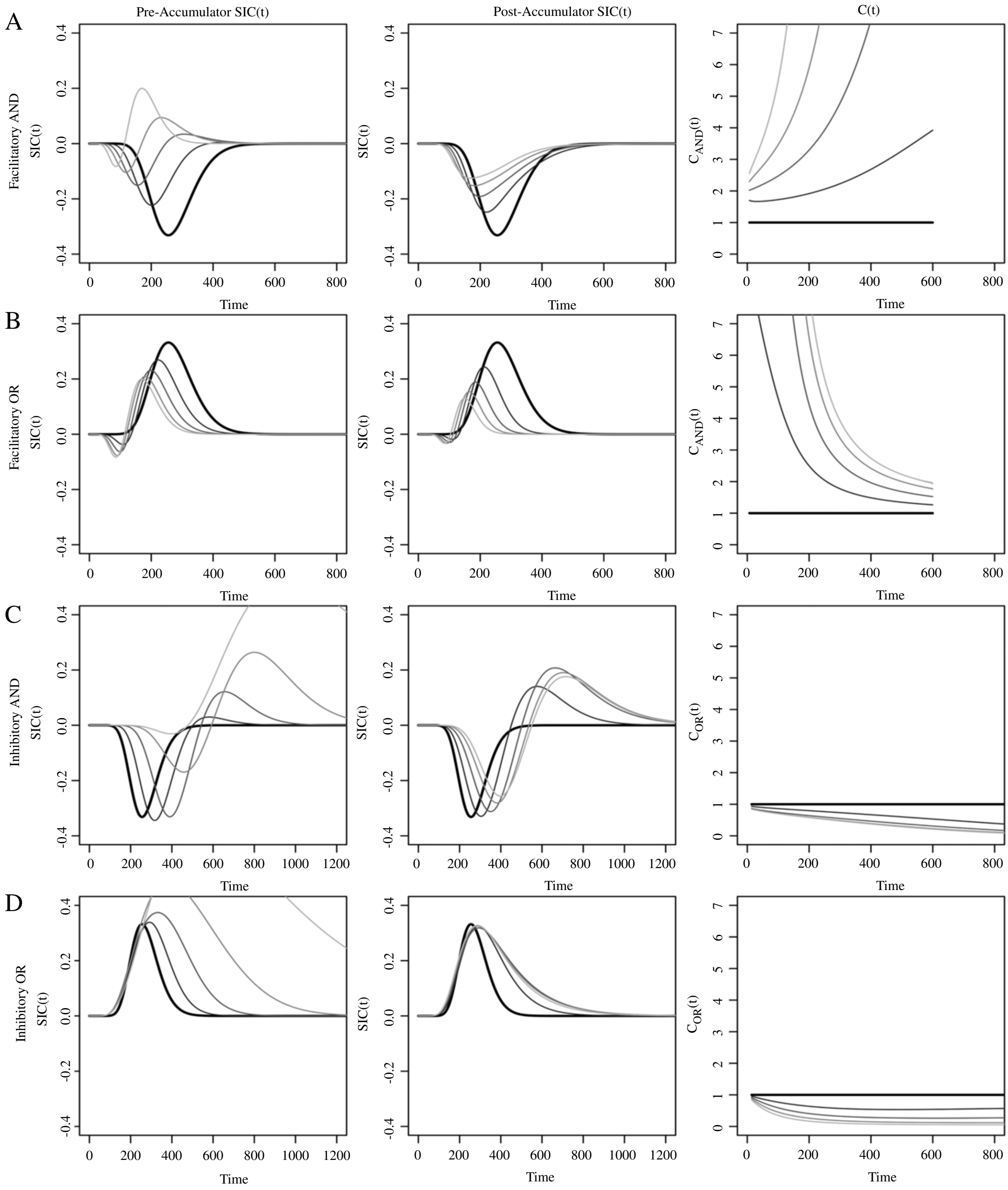

Fig. 3. Simulated SIC results from four types of pre-accumulator parallel interactive models (first column) and post-accumulator models (second column): Facilitatory AND (panel A), Facilitatory OR (panel B), Inhibitory AND (panel C), and Inhibitory OR (panel D). The third column shows the predicted $C(t)$ values, which are similar for pre- and post-accumulator interaction models. Within each panel, the shading of the different lines represents the degree of interaction: the thick dark line represents the independent model and as the probability of interaction increases, the lines become lighter. The probabilities shown here are 0, 0.25, 0.5, 0.75 and 1.

maximum value, the SIC was mostly positive with an early negative blip, again the signature of a coactive model (cf. Fig. 2C).

Regardless of the termination rule then, perfect sharing of counts between channels is structurally identical to coactive processing. The SIC signatures of the two facilitatory models are therefore bounded (from opposite directions) by the SIC signature of the coactive model. This observation is of extreme importance as it allows the researcher to reject certain classes of models. The facilitatory-first-terminating (OR) model, for example, predicted a range of SIC functions that span a finite range from total positivity to mostly positive with an early negative region (Fig. 3B). If an entirely negative SIC function is observed in experimental data, facilitatory first-terminating models can be safely rejected.

Next, consider the forms of the SIC functions produced by parallel-inhibitory models. For the OR case (Fig. 3D), the SIC functions were always positive regardless of the probability of cross-channel interaction. Increasing the level of interaction resulted in an overall slowdown of processing, as demonstrated by the horizontal stretching of the SIC function for high levels of interaction (to the extent that the SIC for the highest level had to be truncated in the figure). However, the qualitative form of the SIC function remained unaffected. Any negativity in the observed SIC rules out inhibitory first-terminating models.

The SIC results of the inhibitory exhaustive (AND) model, in Fig. 3C, pose a more serious challenge for interpretation. The SIC was entirely negative for independent processes, while the right tail gradually became positive as the level of interaction increased. For large amounts of interaction, the positive area exceeded the negative area, and with the maximum amount of interaction the function was almost entirely positive.

Post-accumulator models and comparisons with predictions of pre-accumulator model

Beginning with the inhibitory OR case (Fig. 3D), the SIC predictions for the pre- and post-accumulator models were qualitatively similar. With increased interaction, the SIC function shifted to the right but always remained positive. Thus, any observed negativity in an empirical SIC function immediately rules out inhibitory first-terminating models, regardless of the level, and locus of interaction.

Next, consider the facilitatory OR case in Fig. 3B. Once again, the qualitative predictions of pre- and post-accumulator models were similar. In the absence of cross-channel interaction, the SIC function was entirely positive. With increased interaction it gradually shifted to the left and was increasingly negative for early processing times. Even for the highest levels of interaction, though, it was mostly positive. Therefore, observing a completely negative SIC function, or even a mostly negative function, excludes the facilitatory first-terminating model, again regardless of the locus of interaction.

For the facilitatory AND case (Fig. 3A), the SIC functions predicted by the pre- and post-accumulator models were slightly different. The pre-accumulator model generated a range of SIC functions, from completely negative when processing in the two channels occurs independently, to mostly positive with an early negative blip when interaction was maximal. The post-accumulator model produced SIC functions which were negative across all tested parameter values, and thus comprised only a subset of the pre-accumulator predictions. Observing a completely positive SIC function rules out the facilitatory exhaustive model regardless of its class.

Finally, the predictions of the inhibitory AND model (Fig. 3C) were somewhat similar across both classes. The SIC function was completely negative for independent processing, and its right tail gradually became positive as we increased the level of interaction. For the pre-accumulator model, the function was almost totally positive for the highest possible level of interaction. This model poses a challenge for interpretation as it predicted a wide range of function forms from totally negative to nearly totally positive. To overcome this problem and in general to increase one's ability to discriminate between models based on observed data, one needs to execute the second branch of systems factorial technology—estimating the capacity coefficient, which we shall discuss shortly.

Summarizing the results, most models predicted a finite range of SIC forms. Observing an empirical SIC function that does not fall within the range predicted by a particular model allows the investigators to reject that model. Nonetheless, certain models had overlapping predictions of the SIC function. For concreteness, suppose that you observe an empirical SIC function which is completely positive for all time t. One can immediately rule out the facilitatory exhaustive model (Fig. 3A), as none of the SIC curves constantly stay above the abscissa, regardless of the level and locus of the interaction. However, the facilitatory first-terminating model (Fig. 3B; for $p = 0$, which is an independent model), the inhibitory exhaustive model (Fig. 3C; for $p = 1$) and the inhibitory first-terminating model (Fig. 3D; for all p values including $p = 0$ which is an independent model) could predict a completely positive SIC function. What methodology can be utilized to distinguish between them? At this point, we shall discuss how workload capacity can help distinguishing between inhibitory, facilitatory, and independent parallel models.

Distinguishing between facilitatory and inhibitory models that have similar SIC forms

The capacity coefficient. Inhibitory, facilitatory, and independent-channels models make different predictions with regard to a measure of processing efficiency that gauges workload capacity. By workload capacity, we refer to the processing efficiency of the system as we increase the load of information by, say, increasing the number of the to-be-processed targets. Townsend and Nozawa (1995) proposed a measure of workload capacity—the *capacity coefficient*. For OR processes, the appropriate version is computed as the ratio between the integrated hazard function of the double target condition (i.e., two targets presented simultaneously) and the sum of the integrated hazard functions of the single target conditions:

$$C_{\mathrm{OR}}(t) = \frac{H_{12}(t)}{H_1(t) + H_2(t)}.$$

If the survivor function is the complement of the cumulative distribution function $S(t) = 1 - F(t)$, and the hazard function is the probability density function over the survivor function, $h(t) = \frac{f(t)}{S(t)}$, then the integrated hazard function, $H(t)$ is the integral of the hazard function from zero to t. The subscripts OR indicate that this index is calculated for the OR task.

Recently, Townsend and Wenger (2004b) developed a comparable capacity index for the AND task, $C_{\mathrm{AND}}(t) = \frac{K_1(t)+K_2(t)}{K_{12}(t)}$, where $K(t)$ is analogous to the integrated hazard function, $H(t)$. If we let $k(t)$ (the reverse hazard function, e.g., Chechile, 2010) be equal to the density over the distribution function, $k(t) = \frac{f(t)}{F(t)}$, then $K(t)$ is defined as the integral of $k(t)$ from zero to t.

The interpretation of the two capacity indices, $C_{\mathrm{OR}}(t)$ and $C_{\mathrm{AND}}(t)$, is comparable. When applied to OR processing, $C_{\mathrm{OR}}(t)$ values of 1 imply that the system has an unlimited capacity, such that processing in a given channel is not affected by the increase in workload due to the increase in the number of targets; i.e., a given channel has the same processing rate whether a target is presented to the other channel or not. Likewise, when applied to AND processing $C_{\mathrm{AND}}(t) = 1$ implies unlimited capacity (although when applying $C_{\mathrm{OR}}(t)$ to the unlimited capacity model with AND processing, $C_{\mathrm{OR}}(t) < 1$ due to the increased latency associated with processing all inputs in the redundant target display).

$C_{\mathrm{OR}}(t)$ values that are below 1 in OR processing situations, suggest that capacity is limited, such that increasing the processing load (e.g., by increasing the number of targets on the display) takes a toll on the performance of each channel. $C_{\mathrm{AND}}(t) < 1$ implies limited capacity when applied to either AND or OR processing.

Finally, if $C(t) > 1$ (for either index) then the system is said to have super-capacity; processing efficiency of individual channels actually increases as we increase the workload.

The capacity coefficient gauges the processing efficiency of the system relative to the performance expected from an unlimited capacity independent parallel model. At the same time it indirectly provides information about architecture and channel (in)dependence. For example, the prediction of a parallel-independent model is, by definition, $C(t) = 1$, whereas a standard serial model roughly predicts $C(t) = 0.5$. The prediction of a parallel model with positive cross-channel interactions is $C(t) > 1$, as is the prediction of a coactive model.[5] Very strong inhibitory cross-channel interactions, in either a parallel or serial mode of

5 Townsend and Wenger (2004b) simulated linear dynamic parallel-interactive models and showed that positive channel interactions have a facilitatory effect on

Fig. 4. A decision tree for parallel-interactive model diagnosis. Given both empirical survivor interaction contrast [SIC(t)] and capacity coefficient [$C(t)$] estimates, one can analyze the diagram from top to bottom to rule out models that fail to predict the observed functions. The decision tree accommodates the models tested in this paper. * A coactive model is a candidate.

processing, may lead to severely limited capacity, such that $C(t) < 0.5$. The more inhibition there is between channels, the slower each channel is relative to its performance in isolation, and this slowdown is reflected in smaller values of the capacity coefficient. Conversely, with more cross-channel facilitation, each channel is faster than it would be in isolation, and the coefficient values increase. Thus, the capacity coefficient provides an indication of the degree of facilitation or inhibition.

Independent models, with different combinations of architecture (serial, parallel) and stopping rule (exhaustive, first terminating) predict unique forms of SIC functions (cf. Fig. 2). When different models predict similar survivor contrasts at least one of the models must have high levels of cross-channel interactions. Examining the SIC and the capacity coefficient in tandem provides (in some cases) a decisive test for the architecture and possible dependencies between the processing channels.

In the third column of Fig. 3 we present, for each of the four models, the predictions of the capacity coefficient for various degrees of cross-channel interaction (based on simulations of the pre-accumulator discrete-state model). Like the SIC plots on the same figure, the black line in each panel represents the function for an independent model and as the probability of interaction increases, the shade gets lighter. Under the assumption of parsimony, we can assume that the same underlying processing system generates the data used for estimating SIC(t) and $C(t)$.[6]

With this assumption in mind, we provide the reader with a decision tree (Fig. 4) based on our simulated results. Given the observed SIC(t) and $C(t)$ patterns, one can use this decision tree to decisively rule out certain models that fail to accommodate the observed pattern. The decision tree is restricted to the models tested in this study. Other models may exist that can exhibit similar SIC(t) and $C(t)$ patterns. Therefore, the decision tree is useful for rejecting unsuitable models, but not for determining what is unequivocally the correct model.

When estimating SIC(t) and $C(t)$ from empirical data, the estimates may be influenced by noise. Bootstrapped confidence intervals (Higgins, 2004, pp. 257–258), are an effective way of determining the uncertainty over shape and location of the functions. It is also possible to test whether the positive or negative parts of the SIC(t) are statistically significant using a generalization of the Kolmogorov–Smirnov test (Houpt & Townsend, 2010).

Choosing the appropriate $C(t)$ formula. Given two different formulas, one for $C_{OR}(t)$ and another for $C_{AND}(t)$, how do we know which one to use for our data? In some cases we know what the stopping rule should be and the appropriate measure

workload capacity ($C(t) > 1$) and that negative interactions have an inhibitory effect of capacity ($C(t) < 1$). An unlimited capacity parallel model without cross-channel interactions ($a_{12} = a_{21} = 0$) produces capacity coefficient values of 1. Notably, coactive models in which activation from each channels is summed together produces extremely super capacity values, higher than those observed in parallel models with positive channel interaction.

[6] That is, the architecture does not change when we estimate these two statistics in a single experiment. For example, if two parallel channels operate independently, then they should be independent whether we use the data to estimate the capacity coefficient or whether we use just a subset of this data to estimate the SIC. Within a given experiment, a processing system of some kind cannot exhibit the signatures of independence on one measure (say, the SIC(t) function which is all positive or all negative) and an interaction signature on the other (say, $C(t)$ values much greater than, or much smaller than 1). In fact, the actual level of interaction (p value, in the case of the discrete activation model) is said to be invariant across the two measures.

is clear. For instance, when exhaustive processing is called for by the instructions of a detection task, a failure to comply with the instructions will lead to a noticeable proportion of errors. The participants must use the appropriate rule (AND) in order to perform accurately, and the appropriate capacity measure should be $C_{AND}(t)$.

If we do not know the stopping rule in advance, the form of the SIC can be helpful in determining the appropriate capacity coefficient. Observing a completely negative SIC for all time t rules out the two candidate OR models (left branch of Fig. 4; compare with SIC predictions of OR models in Fig. 3); in this case, the appropriate capacity coefficient would be $C_{AND}(t)$. $C_{AND}(t)$ values greater than 1 rule out inhibitory and independent models, $C_{AND}(t)$ values less than 1 rule out facilitatory and independent models, and $C_{AND}(t) = 1$ is only predicted by independent models.

When the SIC does not provide enough evidence to determine the stopping rule then it is best to choose the most informative version of the capacity coefficient. For example, when the SIC is positive for all t (middle branch of Fig. 4), then we cannot determine the stopping rule. Both inhibitory AND and OR models predict $C_{OR}(t) < 1$, while facilitatory OR models predict $C_{OR}(t) > 1$ but not necessarily $C_{AND}(t) > 1$. Thus $C_{OR}(t)$ is more informative in this case.[7]

Other SIC forms are possible, but they are not predicted by the type of models investigated here. For example, an $\text{SIC}(t) = 0$ for all t is predicted by serial processing models that stop after the first channel is completed (Serial-OR). In some cases, when there is strong asymmetry between channel rates and level of sharing, the currently investigated interactive parallel models can predict other SIC forms as well. These cases are discussed in Appendix C.

In conclusion, models that predict the same form of survivor contrast may be distinguished by observing their $C(t)$ predictions (and vice versa). Within the restricted universe of parallel-interactive models tested here, and given experimentally observed SIC and the capacity coefficient functions in tandem, one can identify a unique candidate processing model (end boxes of each of the paths in Fig. 4). There are only two non-unique cases, but even in these paths the decision tree ends in two candidate models instead of many.[8]

5. Conclusions

In this study, we explored SIC predictions of several classes of interactive parallel models: models with either discrete or continuous activation states, where the locus of interaction can be either pre-accumulation or post-accumulation. For each class, we simulated facilitatory and inhibitory models with OR (inclusive disjunctive) and AND (conjunctive) stopping rules, and generated SIC functions for various levels of cross-channel interactions.

The SIC as a tool for identifying the architecture of underlying processing systems was first introduced by Townsend and Nozawa (1995). These researchers showed that different processing models predict distinctive shapes of the SIC function. Thus, by estimating the SIC directly from data, one can rule out models that fail to predict the observed shape of the contrast function. Townsend and Nozawa limited their exploration to processing models with independent channels. Townsend and Wenger (2004b) studied parallel models with interactions, but focused solely on workload capacity using linear dynamic systems. In this paper we provided a theoretically important generalization of the results of Townsend and colleagues by investigating the SIC predictions of parallel models with cross-channel interaction.

Two important types of parallel models were scrutinized in this paper: discrete-state and continuous-state models. The discrete-state model was constructed as a two channel counting model, in which the input to each channel was a series of single counts that arrived independently, at random intervals, with a constant rate. Each input count was shared, or sent to the other channel, with some fixed probability. The continuous-state model, on the other hand, was formulated as a set of linear differential equations with additive noise. There were no qualitative differences between the results of the discrete- and continuous-state models. This facet is intriguing because it suggests that the feedback mechanism inherent in our stable linear systems modeling may not play a major role in predictions for interactive parallel models.

Using both continuous-state models and discrete-state models, we modeled the effects of pre- and post-accumulation interaction between channels on the form of the SIC function. Despite differences in the formulation of the models, their results were very similar as we demonstrated in Fig. 3.

Although we explored a wide range of parallel interactive models, they predicted a limited range of SIC forms, thereby allowing for the falsification of certain model architectures. Even in the case where different models predict identical SICs, Systems Factorial Technology still provides powerful non-parametric methods for distinguishing among the models. Every pair of facilitatory and inhibitory models that share the same SIC, for instance, can be distinguished by analyzing their capacity predictions. Therefore, combined analysis of empirical SICs and capacity coefficients appears highly promising as a useful experimental tool in model diagnosis. The inferential process is demonstrated in the decision tree shown in Fig. 4.

Under the assumption of selective influence, systems factorial technology has previously shown itself to be a formidable modeling technique. It relies on analytically proven theorems without making parametric assumptions about the underlying distributions responsible for generating the data. As such, its predictions are general and hold for any type of processing model with a particular architecture and stopping rule, regardless of the exact way in which individual channels of the model accumulate evidence over time. For example, a two-channel parallel-independent model always predicts a completely positive SIC, whether the accumulation of evidence towards decision within a channel is based on a diffusion process (e.g. Ratcliff, 1978) or a Poisson process (e.g. Smith & Van Zandt, 2002).

As observed in the Introduction, an earlier theoretical investigation of the consequences of the failure of selective influence on mean interaction contrast led to serious damage to model tests (Townsend & Thomas, 1994). However, that effort, in addition to the confinement to mean interaction contrast, was based on interaction among the actual processing times rather than through state space interactions as we employed above. The theoretical results found that, given no restraints at all on interactive mechanisms that destroy selective influence, mean interaction contrasts can exhibit over additivity, under additivity, or simple additivity. In fact, whether the stochastic interaction among times was positive (e.g., long times in one channel associated with long times in another) or negative (long times in one channel associated with short channels in another), made no difference—each variety could produce the various mean interaction contrast signs.

---

7 In some cases both $C_{AND}(t)$ and $C_{OR}(t)$ are informative such as when the SIC function is negative for early times and positive for late times (Fig. 4, right branch). In this case, one may need to calculate both indices. If $C_{AND}(t) > 1$ then inhibitory AND models can be rejected, leaving both facilitatory models. If, additionally, $C_{OR}(t) < 1$ then facilitatory OR models are rejected in favor of facilitatory AND models.

8 It is important to note that the conclusions one would make from this tree are based on the true $\text{SIC}(t)$ and $C(t)$ functions. In practice one must always estimate these functions from data and thus are subject to sampling error. The fact that each branch of the tree depends on multiple decisions compounds the effect of that error. Furthermore, while statistical tests are available for the SIC (Houpt & Townsend, 2010), bootstrapping is necessary for testing the $C(t)$.

The present approach is unable to assess the SIC predictions of all possible models which violate selective influence. However, our results do suggest optimism that broad, but principled models of processing, founded on general and reasonable state spaces, may make rather generic and canonical predictions. In fact, they indicate that distinct types of interactive-parallel models produce typical signatures (or a limited range of signatures), even when selective influence is violated in specified ways. And, when the empirical survivor contrast and capacity coefficient functions are different from the predicted signatures, certain classes of models that fail to produce the observed outcome can be safely rejected.

Acknowledgments

We would like to express our gratitude to Richard Chechile, Ehtibar Dzhafarov, and Janne Kujala for their helpful comments. We would also like to thank Mario Fific for his insightful ideas. This research was supported by NIH-NIMH MH 057717-07 and AFOSR FA9550-07-1-0078 grants to J. T. T., and by the Keats Endowment grant to A. E.

Appendix A. Formal description of discrete-state models

In this appendix we present the formal description of the discrete-state models discussed in the text. Processing channels in these models simultaneously (i.e., in parallel) accumulate evidence, in form of counts, toward some threshold. Via cross-channel interaction channels can also receive counts from each other. Therefore, counts in each channel could be from two sources: (i) *Within* channel counts, based on the channel's response to some external stimulus or stimulus attribute, that represent the channel independent process of accumulating evidence from the environment. (ii) *Shared* counts, received from the other channel, that represent the interaction across channels.

Modeling within- and between-channel counts

Within channel counts. In both pre- and post-accumulator models, the input counts are assumed to arrive independently and at a constant rate until the channel stops processing. The number of counts accumulated within each channel up to time t, is denoted by $u_1(t)$ and $u_2(t)$ for channels 1 and 2 respectively. To model the difference between high and low salience conditions, the rate for the high condition (H), and thus the probability of accumulating a count in an interval, was set to be higher than the rate for the low condition (L), implying a shorter processing time for the H condition. For the results reported in the main text, the high rate was set to 0.05 and the low rate was set to 0.03 for both channels (but see Appendix C where we report results with a wide range of parameter values).

Between channel counts. The level of interaction between channels is set by the probability p_{12} of channel 1 receiving a count from channel 2 and probability p_{21} of channel 2 receiving a count from channel 1. In the pre-accumulator models, each new within-channel count is shared with a certain probability. We use $k_{ij}(t)$ to denote the number of counts that channel i received from channel j by time t.

In the post-accumulator models, the sharing follows a pure birth process, in which channel i receives counts from channel j according to an exponential distribution with rate $\mu_{ij}(t) = x_j(t)p_{ij}\mu$. $x_j(t)$ is the total activation (shared and within channel) in channel j at time t. p_{ij} is a probability that is varied to model degrees of interaction. The variable μ, with no subscripts, is a constant rate that is independent of the degree of interaction or direction of sharing. In general the sharing rate can be set to any positive number and it does not affect the qualitative aspects of the SIC. For the purposes of this paper, we set it to be in a similar range as the input rate.

Whether count sharing (cross-channel interaction) happens before or after the accumulation of counts, in the facilitatory models the shared count is added to the total activation of the receiving channel so the total activation at time t is the sum of the accumulated within-channel counts and the accumulated shared counts, $x_i(t) = u_i(t) + k_{ij}(t)$. In the inhibitory models, the shared counts are subtracted rather than added. The total activation $x_i(t)$ is then the total accumulated within-channel counts $u_i(t)$ minus the shared counts, $x_i(t) = u_i(t) - \sum_{t' \in \Omega} \Delta k_{ij}(t')$; $\Omega = \{t' | 0 < x_i(t') < \gamma_i; 0 \le t' \le t\}$, where Ω ranges over positive times for which x_i is above zero and below criterion (if activation is zero, or if that channel had reached its criterion, then the shared counts bear no effect).

A channel completes processing when the total activation reaches threshold γ_i. If the system is an OR system, then the system also finishes processing at this point. If it is an AND system, then the other channel will continue *unaffected* by the completed channel. A channel is assumed to have between 0 and γ_i counts, so the model is defined over the $\gamma_1 \times \gamma_2$ state space.

The cumulative distribution function for the AND rule is given by $P_{AND}(RT \le t) = P\{T_1 \le t \text{ AND } T_2 \le t\}$, and the distribution for the OR rule is given by $P_{OR}(RT \le t) = P\{T_1 \le t \text{ OR } T_2 \le t\}$, where T_1 and T_2 are the random variables for processing times on the two channels. The probability that a channel finished processing at or before time t is equivalent to the probability that the total number of counts in the channel is at or above its criterion. Consequently, the cumulative distribution function for the AND rule can also be written as $P_{AND}(RT \le t) = P\{X_1(t) \ge \gamma_1 \text{ AND } X_2(t) \ge \gamma_2\}$ and the distribution for the OR rule is given by $P_{OR}(RT \le t) = P\{X_1(t) \ge \gamma_1 \text{ OR } X_2(t) \ge \gamma_2\}$.

The above discrete-state models are all Markov processes and thus can be analyzed using the general tools associated with that class of models. In particular, we can use a matrix, **R**, of the transition rates to specify the model and to calculate the distribution of completion times. Formally, the transition rate matrix is defined as follows. Suppose v_i is the rate at which the state changes from state i, and q_{ij} is the transition rate from state i to state j.[9] Then the entries of the transition rate matrix are given by $r_{ij} = \begin{cases} q_{ij} & \text{if } i \ne j \\ -v_i & \text{if } i = j \end{cases}$. If $P_{ij}(t) = P\{X(t) = j | X(0) = i\}$, then the matrix of probabilities with entries P_{ij} can be approximated by the equation,

$$\mathbf{P}(t) \approx (\mathbf{I} + \mathbf{R}t/n)^n \tag{A.1}$$

for large n (Ross, 1996, p. 250). The only difference between the models is in the specification of the transition rate matrix.

Facilitatory models

Facilitatory exhaustive (AND) model. Fig. A.1 illustrates the state space for such a model. The state of the model in the figure is represented by the number of counts on channel 1 (y axis) and the number of counts on channel 2 (x axis). The model starts without any counts, at [0, 0], and gradually accumulates evidence towards the thresholds γ_1 and γ_2, thus moving in the state space up and right towards the bounds. At each point of time, the state of the model must fall within one of the five areas in the figure. At any given time t, we are only concerned with the probability that the model has completed, not the exact manner in which it completed. Hence, we can partition the sample space based on the within channel counts, then determine the probability that the model has completed within that partition. This partition is depicted in Fig. A.1. A pre-accumulator model cannot complete processing if its state is within area 5 of Fig. A.1, as there are not enough counts to reach either criterion. However, there is a positive probability

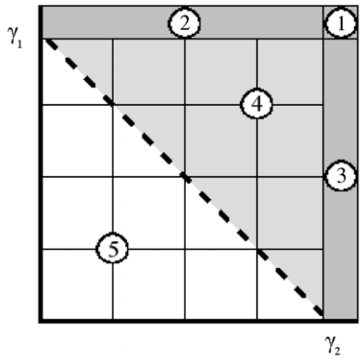

Fig. A.1. The state space of within-channel activation of the discrete state, pre-accumulator models. The y axis corresponds to the level of within-channel activation in channel 1 while the x axis corresponds to channel 2. Area 1 represents the case in which Facilitatory AND and OR models have completed processing. In areas 2 and 3, Facilitatory OR models have terminated and Facilitatory AND models may be finished if there is enough between-channel sharing. In area 4, Facilitatory AND and OR models may finish, but only with enough sharing. In area 5, the pre-accumulator models cannot finish processing, regardless of the amount of sharing.

of completion for the facilitatory AND model in each of the other partitions, areas 1 through 4 in Fig. A.1.

To use Eq. (A.1) for this model, the following equations give the entries for transition rate matrix:

$$v_{(x_1,x_2)(x_1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1;\ x_2 = \gamma_2 \\ \lambda_i & \text{if } x_j = \gamma_j \\ \lambda_1 + \lambda_2 & \text{otherwise,} \end{cases}$$

$$q_{(x_1,x_2)(x_1+1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \\ \lambda_1 & \text{if } x_1 \neq \gamma_1 \text{ and } x_2 = \gamma_2 \\ (1-p_{21})\lambda_1 & \text{otherwise,} \end{cases}$$

$$q_{(x_1,x_2)(x_1,x_2+1)} = \begin{cases} 0 & \text{if } x_2 = \gamma_2 \\ \lambda_2 & \text{if } x_2 \neq \gamma_2 \text{ and } x_1 = \gamma_1 \\ (1-p_{12})\lambda_2 & \text{otherwise,} \end{cases}$$

$$q_{(x_1,x_2)(x_1+1,x_2+1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ p_{21}\lambda_1 + p_{12}\lambda_2 & \text{otherwise.} \end{cases}$$

In the post-accumulator model, any of the counts acquired so far may be shared. Hence, the rate of transition increases as the number of counts increase. The corresponding entries in transition rate matrix are:

$$v_{(x_1,x_2)(x_1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ and } x_2 = \gamma_2 \\ \lambda_i & \text{if } x_i \neq \gamma_i \text{ and } x_j = \gamma_j \\ \lambda_1 + \lambda_2 + \mu(p_{12}x_2 + p_{21}x_1) & \text{otherwise,} \end{cases}$$

$$q_{(x_1,x_2)(x_1+1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \\ \lambda_1 & \text{if } x_1 \neq \gamma_1 \text{ and } x_2 = \gamma_2 \\ \lambda_1 + \mu p_{12}x_2 & \text{otherwise,} \end{cases}$$

$$q_{(x_1,x_2)(x_1,x_2+1)} = \begin{cases} 0 & \text{if } x_2 = \gamma_2 \\ \lambda_2 & \text{if } x_2 \neq \gamma_2 \text{ and } x_1 = \gamma_1 \\ \lambda_2 + \mu p_{21}x_1 & \text{otherwise.} \end{cases}$$

Facilitatory first-terminating (OR) model. In a first-terminating model, only one channel must reach criterion ($u_1 + k_{12} = \gamma_1$ or $u_2 + k_{21} = \gamma_2$). Equivalently, this can be stated as the complement of 'both channels are less than criterion'.

$$P(T_1 < t \text{ OR } T_2 < t) = P(X_1(t) = \gamma_1 \text{ OR } X_2(t) = \gamma_2)$$
$$= 1 - P(U_1(t) + K_{12}(t) < \gamma_1 \text{ AND } U_2(t) + K_{21}(t) < \gamma_2). \quad \text{(A.2)}$$

The transition rate matrices representing the pre- and post-accumulator, facilitatory OR models are quite close to the corresponding matrices for the facilitatory AND models. The only difference is that once either one of the channels has reached its criterion, the transition rate is zero. For the pre-accumulator model:

$$v_{(x_1,x_2)(x_1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_1 + \lambda_2 & \text{otherwise,} \end{cases}$$

$$q_{(x_1,x_2)(x_1+1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ (1-p_{21})\lambda_1 & \text{otherwise ,} \end{cases}$$

$$q_{(x_1,x_2)(x_1,x_2+1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ (1-p_{12})\lambda_2 & \text{otherwise,} \end{cases}$$

$$q_{(x_1,x_2)(x_1+1,x_2+1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ p_{21}\lambda_1 + p_{12}\lambda_2 & \text{otherwise.} \end{cases}$$

For the post-accumulator model:

$$v_{(x_1,x_2)(x_1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_1 + \lambda_2 + \mu(p_{21}x_1 + p_{12}x_2) & \text{otherwise,} \end{cases}$$

$$q_{(x_1,x_2)(x_1+1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_1 + \mu p_{12}x_2 & \text{otherwise,} \end{cases}$$

$$q_{(x_1,x_2)(x_1,x_2+1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_2 + \mu p_{21}x_1 & \text{otherwise.} \end{cases}$$

Inhibitory models

In an inhibitory model the shared counts are subtracted from the total activation of the receiving channel. An additional assumption of the inhibitory models is that the total activation of a channel cannot go below zero (cf. Usher & McClelland, 2001). Such an assumption is not necessary in facilitatory models, because channels' activation cannot be negative. Since the shared counts do not always contribute to the total activation the inhibitory model cannot be stated with the relatively simple equations of the pre-accumulator facilitatory model. Instead we use a random walk process to describe the inhibitory model. We begin by illustrating the state space and possible processing steps in such models (Fig. A.2). In keeping with the intended Poisson nature of the model, we treat the probability of two counts occurring in the same minuscule time increment as zero.

Panel A in Fig. A.2 depicts the initial state of the model and the possible transitions from that state. Initially, there is no activation in either channel so the model starts at [0, 0]. When a channel gains a count, it may or may not share that count. We assume that a channel cannot have negative activation, so if a channel with zero counts receives a shared count, the shared count will have no effect. Therefore, at [0, 0] a count that is added to one channel is not subtracted from its counterpart, and the new state of the model is [1, 0], or [0, 1].

If both channels have at least one count, but neither channel has completed processing, there exist other possible transitions, as depicted in Fig. A.2—Panel B. The model can stay in the same state, both channels could increase, or one channel could increase while the other remains constant.

It is impossible for both channels to lose a count simultaneously. It is also impossible for one channel to lose a count while the other stays the same. This is because for a channel to lose a count, it must receive a shared count from the other channel and not gain a within-channel count. Since the first channel does not gain a within-channel count, it cannot share. However, the other channel must gain a within-channel count to share. This other channel cannot have received a shared count from the first channel, meaning that its total activation must also increase. Thus, for one channel to decrease, the other must increase.

Once a channel reaches criterion, the behavior of the first-terminating and the exhaustive models diverges. The first-

[9] Note that here i and j refer to the state, not to indicate specific channels as is the case above.

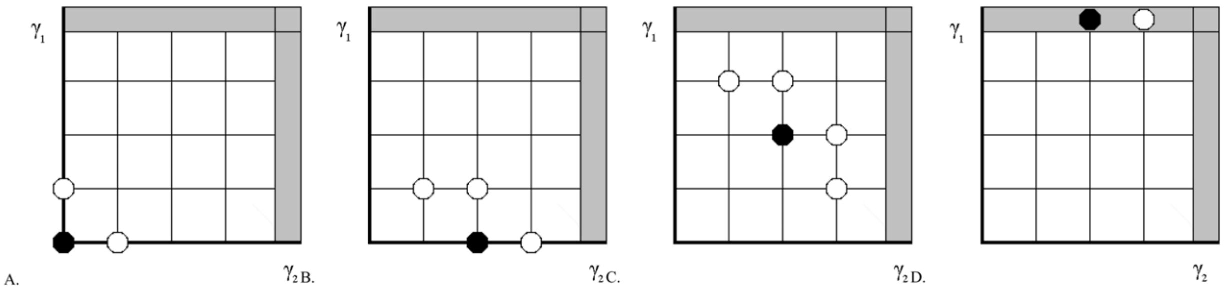

Fig. A.2. The state space of total channel activation of inhibitory discrete-state models. The *y* axis corresponds to the level of activation in channel 1 and the *x* axis corresponds to channel 2. If the model is in the state marked by the black dot, then the possible states in the next time step are depicted by all of the dots, including the possibility of staying in the current state. Panel A shows the initial state of the model. Panel B shows an example of a state in which one channel has acquired some activation while the other has none. Panel C shows an example of a state in which both channels have some activation, but neither has reached its criterion. Panel D shows an example of a state in which one channel has reached criterion but the other has not. If the model is an OR model, processing has terminated. If it is an AND model, then only the activation in the channel that is still below criterion can change.

terminating model finishes processing at this point (so the model will not transition to any other state). The exhaustive model must continue processing until both channels reach criterion. After one channel completes processing it can no longer affect processing in the other channel. The unfinished channel will continue accumulating counts independently until it reaches its criterion. When both channels reach criteria the model reaches its final state (and processing is completed).

We are now in a position to specify the transition rate matrix for these models. In most cases, the transition rate depends on the current state. Cases where at least one channel is zero or at criterion, pictured in Fig. A.2—Panels A, B and D, are dealt with first, then we specify transition rates from states in which both channels have at least one count and neither channel has reached criterion. These are the states exemplified in Fig. A.2—Panel C. As we described above, the model can transition to a state where only a single channel increased while the other channel either stayed the same or decreased, or to the same state in which both channels have the same amount of activation.

Inhibitory exhaustive (AND) model. In the AND model, the unfinished channel will continue independently until it finishes.

For one channel to increase while the other decreases, the first channel must gain a count, then share it.

$$q_{(x_1,x_2)(x_1+1,x_2-1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1,\ x_2 = \gamma_2 \text{ or } x_2 = 0 \\ p_{21}\lambda_1 & \text{otherwise} \end{cases}$$

$$q_{(x_1,x_2)(x_1-1,x_2+1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1,\ x_2 = \gamma_2 \text{ or } x_1 = 0 \\ p_{12}\lambda_2 & \text{otherwise.} \end{cases}$$

For the total activation in one channel to increase while the other remains the same, the first channel must obtain a within-channel count but not share it.

$$q_{(x_1,x_2)(x_1+1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \\ \lambda_1 & x_2 = \gamma_2 \text{ or } x_2 = 0 \text{ and } x_1 \neq \gamma_1 \\ (1-p_{21})\lambda_1 & \text{otherwise} \end{cases}$$

$$q_{(x_1,x_2)(x_1,x_2+1)} = \begin{cases} 0 & \text{if } x_2 = \gamma_2 \\ \lambda_2 & x_1 = \gamma_1 \text{ or } x_1 = 0 \text{ and } x_2 \neq \gamma_2 \\ (1-p_{12})\lambda_2 & \text{otherwise.} \end{cases}$$

If neither channel gains a count then no counts had been shared and the activation simply stays the same.

$$v_{(x_1,x_2)(x_1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ and } x_2 = \gamma_2 \\ \lambda_i & \text{if } x_i \neq \gamma_i \text{ and } x_j = \gamma_j \\ \lambda_1 + \lambda_2 & \text{otherwise.} \end{cases}$$

The post accumulator models are similar to facilitatory post-accumulator models. The diagonal entries to the transition rate matrix are the same.

$$v_{(x_1,x_2)(x_1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ and } x_2 = \gamma_2 \\ \lambda_i & \text{if } x_i \neq \gamma_i \text{ and } x_j = \gamma_j \\ \lambda_1 + \lambda_2 + \mu(p_{21}x_1 + p_{12}x_2) & \text{otherwise.} \end{cases}$$

The difference is that when a count is shared, the receiving channel decreases. Hence, the transition rates for gaining a count in a channel are,

$$q_{(x_1,x_2)(x_1+1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \\ \lambda_1 & \text{otherwise} \end{cases} \quad \text{and}$$

$$q_{(x_1,x_2)(x_1,x_2+1)} = \begin{cases} 0 & \text{if } x_2 = \gamma_2 \\ \lambda_2 & \text{otherwise.} \end{cases}$$

The transition rates for losing a count are,

$$q_{(x_1,x_2)(x_1,x_2-1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \mu p_{12}x_2 & \text{otherwise.} \end{cases} \quad \text{and}$$

$$q_{(x_1,x_2)(x_1-1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \mu p_{21}x_1 & \text{otherwise.} \end{cases}$$

Inhibitory first-terminating (OR) model. The transition probabilities listed up to this point apply to both the OR and AND models. As discussed earlier, the two models differ once one of the channels finishes. In this case, the OR model does not change states. In all other cases, the behavior of the two models is identical.

For one channel to increase while the other decreases, the first channel must gain a count, then share it.

$$q_{(x_1,x_2)(x_1+1,x_2-1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1, x_2 = \gamma_2 \text{ or } x_2 = 0 \\ p_{21}\lambda_1 & \text{otherwise} \end{cases}$$

$$q_{(x_1,x_2)(x_1-1,x_2+1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1, x_2 = \gamma_2 \text{ or } x_1 = 0 \\ p_{12}\lambda_2 & \text{otherwise.} \end{cases}$$

For the total activation in one channel to increase while the other remains the same, the first channel must obtain a within-channel count but not share it.

$$q_{(x_1,x_2)(x_1+1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_1 & \text{if } x_2 = 0 \text{ and } x_1 \neq \gamma_1 \\ (1-p_{21})\lambda_1 & \text{otherwise} \end{cases}$$

$$q_{(x_1,x_2)(x_1,x_2+1)} = \begin{cases} 0 & \text{if } x_2 = \gamma_2 \text{ or } x_1 = \gamma_1 \\ \lambda_2 & \text{if } x_1 = 0 \text{ and } x_2 \neq \gamma_2 \\ (1-p_{12})\lambda_2 & \text{otherwise.} \end{cases}$$

If neither channel gains a count then no counts had been shared and the activation simply stays the same.

$$v_{(x_1,x_2)(x_1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_1 + \lambda_2 & \text{otherwise.} \end{cases}$$

Like the post-accumulator AND models, the post-accumulator inhibitory OR models are quite similar to the post-accumulator facilitatory OR models,

$$v_{(x_1,x_2)(x_1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_1 + \lambda_2 + \mu(p_{21}x_1 + p_{12}x_2) & \text{otherwise,} \end{cases}$$

$$q_{(x_1,x_2)(x_1+1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_1 & \text{otherwise,} \end{cases}$$

$$q_{(x_1,x_2)(x_1,x_2+1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_2 & \text{otherwise} \end{cases}$$

$$q_{(x_1,x_2)(x_1-1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \mu p_{12}x_2 & \text{otherwise} \end{cases}$$

$$q_{(x_1,x_2)(x_1,x_2-1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \mu p_{21}x_1 & \text{otherwise.} \end{cases}$$

Appendix B. Formal description of continuous-state models

In this appendix we present the formal description of the continuous-state models discussed in the text. Like the discrete-state models in Appendix A, we assume two parallel processing channels, but now we allow the state to be any positive real number, as opposed to just integer value. The total activation in each channel is represented by $x_i(t)$, although to conform to the standard presentation of linear dynamic systems (e.g. Townsend & Wenger, 2004b), we use vector and matrix notation, i.e. $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$. Each channel has some input, $\mathbf{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$, corresponding to the within-channel counts in the discrete-state model.

To represent cross-channel interactions, we use a matrix of coefficients indicating the values of the activation weights. Following Ashby's model for stochastic general recognition theory (Ashby, 1989), we use $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ for pre-accumulator interactions, and $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ for post-accumulator interactions. The off-diagonal coefficients represent the amount of between-channel cross talk, or information sharing, so a_{12} determines the amount of cross-talk received by channel 1 from channel 2 and a_{21} determines the amount of cross-talk received channel 2 from channel 1. For those unfamiliar with linear dynamic system notation, it may seem odd to use a_{12} for the sharing from channel 2 to channel 1 rather than vice versa. In keeping with the standard notation of this class of models, we use the subscripts to denote the row and column of the matrix **A**.

By setting the off-diagonal coefficients of matrix **A** to zero, cross-channel sharing is completely eliminated, thereby making the model equivalent to an independent-parallel model. Activation in the model is then solely dependent on the diagonal elements, representing within-channel contribution. The diagonal elements b_{11} and b_{22} are parameters denoting gain or loss applied to the within channel input. Since changing the diagonal elements of **B** is equivalent to rescaling the inputs, we fixed them to 1, $b_{11} = b_{22} = 1$. The diagonal elements a_{11} and a_{22} are parameters denoting the feedback rate for a particular channel. As we shall see shortly, these values can be used to ensure that the system is stable. Townsend and Wenger (2004b) used parameter values that maintained stability in the system, a property that is often assumed for natural systems (cf. Usher & McClelland, 2001).

Deterministic pre-accumulator model

The two-channel pre-accumulator interactive parallel model, with no post-accumulator interaction, is given by:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}. \quad \text{(B.1)}$$

We refer to the above version of the model as *deterministic*, because it has no source of noise or variability. We shall shortly present the *stochastic* version of the model, which includes a noise term.

The magnitude of the interaction parameters (off diagonal elements of **B**) was varied between 0 and 1 to represent the range between complete independence and total information sharing. Similar to our explorations with the discrete-state models, we set the interaction to be symmetric so that $b_{12} = b_{21}$. Assuming a constant input, the solution to this differential equation is

$$x_1(t) = \frac{u_1 + b_{12}u_2}{a_{11}}[\exp(a_{11}t) - 1]$$
$$x_2(t) = \frac{b_{21}u_1 + u_2}{a_{22}}[\exp(a_{22}t) - 1]. \quad \text{(B.2)}$$

Deterministic post-accumulator model

The (deterministic) two-channel post-accumulator interactive parallel model is given by:

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{u}(t) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}. \quad \text{(B.3)}$$

In accordance with Townsend and Wenger (2004b), we further simplified the model with the assumption that the activation rates within each channel are equal, $a_{11} = a_{22}$, and as above, cross-channel interaction coefficients are equal, $a_{12} = a_{21}$. Furthermore, we assumed that the input to each channel is constant (for $t > 0$, $u_1(t) = u_1$; $u_2(t) = u_2$), making the system time invariant.

In this case there exists a closed form solution that describes the activation level in each of the channels at time $t \geq 0$:

$$x_1(t) = \frac{u_1 + u_2}{2(a_{11} + a_{12})}[\exp[(a_{11} + a_{12})t] - 1] + \frac{u_1 - u_2}{2(a_{11} - a_{12})}[\exp[(a_{11} - a_{12})t] - 1]$$
$$x_2(t) = \frac{u_1 + u_2}{2(a_{11} + a_{12})}[\exp[(a_{11} + a_{12})t] - 1] + \frac{u_2 - u_1}{2(a_{11} - a_{12})}[\exp[(a_{11} - a_{12})t] - 1]. \quad \text{(B.4)}$$

The channel's activation is an exponential expression, meaning that if the sum $a_{11} + a_{12}$ or $a_{11} - a_{12}$ is positive, the activation increases without bound. To stabilize the system, we set $a_{11} = a_{22} < 0$; and $|a_{12}| = |a_{21}| < |a_{11}| = |a_{22}|$ to prevent the sum from being positive.

Stochastic pre- and post-accumulator models

To make the model stochastic, we added a two-dimensional Brownian motion process, $\mathbf{W}(t)$, to the input. The added noise process is independently and identically distributed over time and across channels. The differential equation that describes channel activation in a stochastic model with two parallel channels that interact *pre-accumulation* is:

$$d\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t)dt + \mathbf{B}[\mathbf{u}(t)dt + d\mathbf{W}(t)] = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}dt + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}dt + \begin{bmatrix} dW_1(t) \\ dW_2(t) \end{bmatrix}. \quad \text{(B.5)}$$

When the interaction occurs *post-accumulation*, the equation is:

$$d\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t)dt + \mathbf{u}(t)dt + d\mathbf{W}(t) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}dt + \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}dt + \begin{bmatrix} dW_1(t) \\ dW_2(t) \end{bmatrix}. \quad \text{(B.6)}$$

Table C.1
Parameters and parameter rates used in simulations of the discrete-state models.

Parameter	Values tested
v_{1h}: High rate, channel 1	0.05
v_{2h}: High rate, channel 2	0.02, 0.03, 0.04, 0.05
v_{1l}: Low rate, channel 1	0.01, 0.02, 0.03, 0.04
v_{2l}: Low rate, channel 2	0.01, 0.02, 0.03, 0.04 (and smaller than v_{2h})
γ_1: Criterion, channel 1	5, 10, 15
γ_2: Criterion, channel 2	5, 10, 15
p_{12}: Probability of sharing received by channel 1 from 2	0, 0.2, 0.4, 0.6, 0.8, 1
p_{21}: Probability of sharing received by channel 2 from 1	0, 0.2, 0.4, 0.6, 0.8, 1

By adding the white noise processes, the derivative can no longer be interpreted as a standard derivative. This follows from the fact that Brownian motion, and hence the processes described in Eqs. (B.5) and (B.6) are nowhere differentiable. Instead, these equations may be interpreted as solutions to an Itô integral. In practice, we did not calculate exact distributions from these equations; we only simulated results using a discrete-time approximation.

As in the discrete-state models, we allowed the interaction parameters to be either positive (facilitation) or negative (inhibition), and manipulated the magnitude of the cross-channel interaction. The actual interaction parameters for the post-accumulator, facilitatory models were set to be 0 (independent channels), 1.2, 2.4 and 3.6 and 4.8, with the stabilizing parameter set to −10. For simplicity, both the interaction parameters and stabilizing parameters were set to be equal across channels ($a_{12} = a_{21}$; $a_{11} = a_{22}$). The particular range of parameter values was chosen to ensure the stability of the model. As stated above, the cross channel interaction in the pre-accumulator models varied in magnitude from 0 to 1, with positive values for facilitatory values and negative values for inhibitory models.

Appendix C. Simulation results from a wide range of parameter values

We have explored SIC predictions of parallel-interactive models over a wide range of parameter values. In the main text and in Fig. 3 we described the SIC forms that were most prevalent in our simulation results (we report below, in Table C.2, the exact frequencies of these SIC patterns, as well as frequencies of other patterns). These forms, which we refer to as the canonical SIC forms, were replicated with many parameter values, as long as the system was roughly symmetrical (i.e., the two channels had roughly the same rates, roughly the same criterion values, and roughly the same level of sharing). When evidence-accumulation processes in the channels were highly asymmetrical (we shall be more precise when we refer to Table C.2), other SIC patterns emerged. In Table C.1 we present the parameters and parameter values tested with the discrete-state models. Results from continuous-state models were qualitatively similar. The results reported in the main text and illustrated in Fig. 3 are based on a subset of this range, where channels were symmetric in rate, criterion, and sharing (namely, high rate was set to 0.05 and low rate was set to 0.03, for both channels; criterion was 5 for both channels; probability of cross-channel interaction varied, but was always equal for both channels).

In Table C.2 we summarize results from simulating the full parameter range. On each simulation we varied the values of each parameter independently of the other parameters, and tested the form of the SIC function for this combination of parameters. We repeated the simulations for Facilitatory AND, Facilitatory OR, Inhibitory AND, and Inhibitory OR models. Overall, we tested 12,960 parameter combinations for each model, amounting to a total of 51,840 simulated SIC functions. In effect, this number was doubled, since we simulated models with both pre- and post-accumulator interactions. The results from the two model types were generally similar, but for completeness we report both sets, in separate columns.

Table C.2
Simulation results from four types of discrete-state models with pre- and post-accumulator sharing, given various combinations of parameter values. The parameter values used for these simulations are listed in Table C.1. PN: Positive then negative. NPN: negative, then positive, then negative (see text for more details).

Model	Pre-accumulator sharing	Post-accumulator sharing
Facilitatory AND	Canonical form: 95.1% Other forms: SIC = 0 for all t: 4.9%	Canonical form: 76.8% Other forms: SIC = NPN: 18.9% SIC = PN: 3.9% SIC = 0 for all t: 0.5%
Facilitatory OR	Canonical Form: 100.0%	Canonical Form: 100.0%
Inhibitory AND	Canonical Form: 99.8% Other forms: SIC = NPN: 0.2%	Canonical Form: 100.0%
Inhibitory OR	Canonical Form: 80.9% Other forms: SIC = PNP: 11.3% SIC = PN: 7.7% SIC = 0 for all t: 0.1%	Canonical Form: 83.6% Other forms: SIC = PNP: 13.3% SIC = PN: 3.1% SIC = 0 for all t: 0.1%

For each model we first report the proportion of cases (each case is a different combination of parameters) in which the SIC function was similar in form to one of the canonical SIC signatures of this model, presented in Fig. 3. Then, for each model, we list the other, non-canonical SIC forms and the proportion of times each of these forms was observed in our simulations. For brevity, when describing the non-canonical forms we use N and P to indicate whether the SIC function was negative or positive. So, for example, PN means that the SIC function started positive and then became negative for larger t, and PNP means that the function started positive, became negative, and then became positive again for large t.

We discuss the simulation results for the various models presented in Table C.2 by order of difficulty for interpretation, from easy to more difficult, and not necessarily by order of presentation. A scrutiny of the table reveals that for a wide range of parameter values and regardless of the locus of interaction, facilitatory OR and inhibitory AND models consistently predict SIC signatures similar to the canonical SIC forms depicted in Fig. 3. The pre-accumulator inhibitory AND model predicted a non-canonical, SIC = NPN with as little as 0.2% of the parameter settings. In each of the cases that predict this form, both the probability of sharing and the criteria were quite high: $p_{12} + p_{21} \geq 1.6$; $\gamma_1 + \gamma_2 \geq 20$.

The pre-accumulator facilitatory AND model predicts canonical SIC forms on more than 95% of the cases, and predicts a non-canonical SIC form, $\mathrm{SIC}(t) = 0$ for all t, only when one channel is faster (i.e., has a faster rate or lower criterion) than the other and the probability of sharing from the faster channel to the other is null whereas the other, slower channel shares with some nonzero probability.

The post-accumulator facilitatory AND model exhibits more variability in its predicted SIC forms. In nearly 19% of cases the SIC had an NPN form. These cases all had some asymmetry in the parameters, however asymmetry in the post-accumulator facilitatory AND models did not necessarily imply the NPN form. When the expected completion time of one channel was much faster than that of the other, SIC = PN form was also occasionally observed. $\mathrm{SIC}(t) = 0$ for all t occurred in similar conditions to the pre-accumulator facilitatory AND model, but was not as common: 0.5% of the parameter settings.

Finally, the predictions of the pre- and post-accumulator inhibitory OR models are well within the range of the canonical SIC forms for more than 80% of the cases we have investigated. These models, like the post-accumulator facilitatory AND model, predict the non-canonical SIC = PN form (positive for small t, then turning negative for larger t) when strong asymmetry is induced: either the expected completion time of one channel is much faster than the other, or the probability of sharing from one channel is relatively small whereas the probability of sharing from the other channel is high. Additionally, both types of the inhibitory OR models very rarely produce $\mathrm{SIC}(t) = 0$ for all t. Much like the facilitatory AND models, these cases only occurred when the probability of sharing from one channel to the other was zero while sharing in the other direction was non-zero. Importantly, the inhibitory OR models are the only models we have tested that predict a non-canonical SIC form (namely, PNP—positive for small t, then turning negative, and finally positive for large t) without postulating channel asymmetry. Luckily, this predicted non-canonical form — PNP — is unique to that model as far as our tests show. That is, observing a PNP signature is a useful diagnostic tool as it rules out the other models (and in any event appears on a small subset, 11% and 13%, of the pre- and post-accumulator simulations, respectively).

In conclusion, repeating the simulations with a wide range of parameter values resulted in only few qualitative changes in the form of the SIC function compared to the canonical forms reported in the main text and in Fig. 3. Allowing the rates, criterion values, and probabilities of sharing to vary independently between channels generally resulted in canonical SIC forms. In a few cases, for specific parameter combinations, strong asymmetry across channels (different rates, criteria, and/or level of cross-channel sharing) may result in non-canonical forms. These non-canonical forms are unique in the sense that they are not predicted by any of the independent models, nor by any of the symmetric interactive-parallel models tested here (for example, no independent model predicts SIC = PN, that is, SIC that starts positive and becomes negative for large t). Therefore, these SIC signatures are easy to identify and are not likely to lead to a false identification of models' architecture.

References

Ashby, F. G. (1989). Stochastic general recognition theory. In D. Vickers, & P. L. Smith (Eds.), *Human information processing: measures, mechanisms and models* (pp. 435–457). Amsterdam: Elsevier Science Publishers.

Chechile, R. (2010). Fundamental properties of reverse hazard functions. In: *Paper presented at the annual meeting of the society of mathematical psychology*.

Donders, F. C. (1868). On the speed of mental processes. *Acta Psychologica*, *30*, 412–431. (W. G. Koster, Trans. 1969).

Dzhafarov, E. N. (2003). Selective influence through conditional independence. *Psychometrika*, *68*, 7–26.

Eidels, A., & Townsend, J. T. (2009). Testing response time predictions of a large class of parallel models within OR and AND redundant signals paradigm. In: *Paper presented at the annual meeting of the society of mathematical psychology*.

Eidels, A., Townsend, J. T., & Algom, D. (2010). Comparing perception of Stroop stimuli in focused versus divided attention: evidence for dramatic processing differences. *Cognition*, *114*, 129–150.

Higgins, J. J. (2004). *Introduction to modern nonparametric statistics*. Pacific Grove: Thomson.

Houpt, J. W., & Townsend, J. T. (2010). The statistical properties of the survivor interaction contrast. *Journal of Mathematical Psychology*, *54*, 446–453.

Kujala, J. V., & Dzhafarov, E. N. (2008). Testing for selectivity in the dependence of random variables on external factors. *Journal of Mathematical Psychology*, *52*, 128–144.

Ratcliff, R. (1978). Theory of memory retrieval. *Psychological Review*, *85*, 59–108.

Ross, S. M. (1996). *Stochastic processes* (2nd ed.). New York: Wiley.

Schweickert, R., & Townsend, J. T. (1989). A trichotomy method: interactions of factors prolonging sequential and concurrent mental processes in stochastic PERT networks. *Journal of Mathematical Psychology*, *33*, 328–347.

Smith, P. L., & Van Zandt, T. (2002). Time-dependent Poisson counter models of response latency in simple judgment. *British Journal of Mathematical and Statistical Psychology*, *53*, 293–315.

Snodgrass, J. G., & Townsend, J. T. (1980). Comparing parallel and serial models: theory and implementation. *Journal of Experimental Psychology: Human Perception and Performance*, *6*, 330–354.

Sternberg, S. (1969). Memory scanning: mental processes revealed by reaction-time experiments. *American Scientist*, *4*, 421–457.

Townsend, J. T. (1972). Some results concerning the identifiability of parallel and serial processes. *British Journal of Mathematical and Statistical Psychology*, *25*, 168–199.

Townsend, J. T. (1984). Uncovering mental processes with factorial experiments. *Journal of Mathematical Psychology*, *28*, 363–400.

Townsend, J. T. (1990a). Serial vs. parallel processing: sometimes they look like tweedledum and tweedledee but they can (and should) be distinguished. *Psychological Science*, *1*, 46–54.

Townsend, J. T. (1990b). Truth and consequences of ordinal differences in statistical distributions: toward a theory of hierarchical inference. *Psychological Bulletin*, *108*, 551–567.

Townsend, J. T., & Ashby, F. G. (1983). *The stochastic modeling of elementary psychological processes*. Cambridge, UK: Cambridge University Press.

Townsend, J. T., & Nozawa, G. (1995). Spatio-temporal properties of elementary perception: an investigation of parallel, serial, and coactive theories. *Journal of Mathematical Psychology*, *39*, 321–359.

Townsend, J. T., & Schweickert, R. (1989). Toward the trichotomy method: laying the foundation of stochastic mental networks. *Journal of Mathematical Psychology*, *33*, 309–327.

Townsend, J. T., & Thomas, R. (1994). Stochastic dependencies in parallel and serial models: effects on systems factorial interactions. *Journal of Mathematical Psychology*, *38*, 1–34.

Townsend, J. T., & Wenger, M. J. (2004a). The serial–parallel dilemma: a case study in a linkage of theory and method. *Psychonomic Bulletin & Review*, *11*, 391–418.

Townsend, J. T., & Wenger, M. J. (2004b). A theory of interactive parallel processing: new capacity measures and predictions for a response time inequality series. *Psychological Review*, *111*, 1003–1035.

Usher, M., & McClelland, J. L. (2001). On the time course of perceptual choice: the leaky competing accumulator model. *Psychological Review*, *108*, 550–592.