Determining bias and power from the simulation results from Heathcote, A., Brown, S. & Mewhort, D. J. K. (in press). Quantile Maximum Likelihood Estimation of Response Time Distributions.

The values in tables A1-A5, based on simulated data with standard deviation of 100, can be mapped to millisecond units using the standard deviation observed in data. If, for example, more variable data with a standard deviation of 200 ms was observed, the bias and standard deviation values in the tables should be multiplied by a factor of two. The use of the tables to check for confounding due to bias, and to perform a power analysis, will be illustrated with two examples.

Bias may produce spurious differences when comparing conditions with different values of K. For purposes of illustration, the following example uses the most extreme case available in the tables, comparing conditions with K = 1/3 and K = 5 with $\underline{n}$ = 40 and 4 observations per inter-quantile interval. The example focuses on bias in μ, as bias for σ estimates was negligible, and σ and τ are assumed to take values that produce the difference in K while maintaining a constant standard deviation in the two conditions.

Assume that there are no real differences in μ between the two conditions. In the simulation data, there were actually differences in μ for the K = 1/3 and K = 5 distributions. However, similar bias results would have been obtained if μ was equivalent and K manipulated as described. From Table A3, bias in μ estimation equals –12.37 units for K = 1/3 and 5.49 units for K = 5. Therefore, on average, a difference in μ estimates

between the two conditions of around 18 units could be due to bias. Note that K may be robustly estimated from data using Table 1 and the distribution asymmetry measure, A.

The tables can also be used to estimate the sample size required in a study examining ex-Gaussian parameter estimates. Suppose a difference of 20 ms in parameter estimates must be detected with samples of 40 observations per subject, one observation per inter-quantile interval, the data that has a standard deviation of 100, and K = 2. Assume that differences between subject K and standard deviation values are negligible. Where this does not hold, appropriate entries in the tables should be averaged in order to estimate the average standard deviation of the parameter sampling distributions. From Table A3, the expected standard deviations of individual parameter estimates are 21 for μ, 15.4 for σ, and 24.6 for τ. Assuming additive differences between subjects across two conditions, the contribution of between-subject variation to error can be ignored. For a two-tailed Z test with critical values of ± 1.96 at $\alpha = 0.05$, and equal numbers of subjects in each condition (ns), the number of subjects required to detect the 20 ms difference is given by: $ns = 2 \times (1.96 \times SD/20)^2$. Using the nearest integer greater than this estimate, a sample of 12 subjects has sufficient power to detect the difference even for the most variable (τ) parameter. Generalizations to t and F tests are straightforward.

Table A1. Bias (Mean and Median) and Efficiency (SD) for one observation per inter-quantile interval

		Mu			Sigma			Tau		
K	n	Mean	Median	SD	Mean	Median	SD	Mean	Median	SD
	40	-7.39	-3.59	29.91	-5.89	-4.04	16.84	7.67	2.46	26.25
1/3	80	-4.11	-1.87	22.78	-3.42	-2.39	11.52	4.34	1.92	20.35
	160	-2.47	-1.53	17.69	-2.22	-1.52	8.15	2.51	1.48	16.23
	40	0.16	2.65	30.90	-3.83	-2.18	17.42	0.13	-2.81	28.11
1/2	80	1.74	2.26	24.06	-1.73	-0.61	12.36	-1.51	-1.58	22.32
	160	2.87	2.03	18.53	-0.35	0.12	8.81	-2.78	-1.33	17.48
	40	5.33	2.65	29.55	-0.60	-0.40	18.06	-4.65	-1.67	29.86
1	80	3.42	1.31	21.52	0.15	-0.08	12.81	-3.11	-0.83	21.73
	160	1.81	0.66	14.94	0.21	-0.08	9.06	-1.70	-0.42	14.87
	40	3.42	1.42	21.01	0.11	-0.42	15.39	-2.48	-1.45	24.64
2	80	1.50	0.64	13.79	0.15	-0.16	10.40	-1.14	-0.83	16.37
	160	0.51	0.20	9.41	-0.04	-0.20	7.08	-0.31	-0.21	11.20
	40	3.33	1.74	16.42	0.59	-0.26	13.29	-2.38	-2.27	21.72
3	80	1.24	0.69	10.71	0.15	-0.13	8.68	-0.78	-0.88	14.63
	160	0.58	0.37	7.30	0.07	-0.01	5.86	-0.32	-0.46	10.15
	40	3.29	1.84	14.17	0.95	-0.00	11.93	-0.20	-0.25	20.29
4	80	1.24	0.71	9.15	0.20	-0.16	7.73	1.40	1.17	13.88
	160	0.58	0.41	6.11	0.10	0.02	5.12	1.86	1.74	9.58
	40	3.47	1.91	12.75	1.33	0.26	11.12	-2.37	-2.68	19.71
5	80	1.43	0.87	8.02	0.38	0.09	6.97	-0.97	-1.25	13.27
	160	0.62	0.36	5.44	0.14	0.04	4.64	-0.39	-0.56	9.32

Table A2. Bias (Mean and Median) and Efficiency (SD) for two observations per inter-quantile interval

		Mu			Sigma			Tau		
K	n	Mean	Median	SD	Mean	Median	SD	Mean	Median	SD
	40	-9.08	-4.88	30.84	-7.03	-4.76	18.03	9.66	2.69	27.72
1/3	80	-4.90	-2.33	23.54	-3.96	-2.78	11.98	5.19	2.20	21.27
	160	-2.82	-1.80	17.91	-2.41	-1.67	8.31	2.89	1.98	16.47
	40	-0.80	2.22	31.56	-4.78	-2.68	18.51	1.27	-3.54	29.29
1/2	80	1.55	2.54	24.67	-2.08	-0.78	12.78	-1.33	-2.35	23.08
	160	2.93	2.43	18.79	-0.43	0.18	8.97	-2.85	-1.98	17.79
	40	6.24	3.55	30.53	-0.99	-0.43	19.38	-5.65	-2.79	31.34
1	80	3.88	1.60	22.17	0.05	-0.09	13.30	-3.67	-1.20	22.66
	160	2.04	0.80	15.11	0.20	-0.04	9.18	-1.93	-0.65	15.23
	40	4.19	1.78	21.95	-0.23	-0.63	16.98	-3.47	-2.02	25.96
2	80	1.70	0.75	14.15	0.04	-0.23	10.95	-1.45	-0.97	16.87
	160	0.56	0.25	9.45	-0.12	-0.26	7.19	-0.40	-0.32	11.34
	40	3.87	1.99	17.07	0.03	-0.59	15.12	-3.22	-2.79	22.61
3	80	1.36	0.78	10.95	0.01	-0.26	9.27	-1.05	-1.07	14.94
	160	0.59	0.35	7.36	-0.02	-0.08	6.04	-0.40	-0.52	10.22
	40	3.77	2.01	14.81	0.36	-0.32	13.88	-1.05	-0.96	21.06
4	80	1.37	0.74	9.31	0.10	-0.22	8.32	1.13	0.95	14.09
	160	0.59	0.41	6.18	0.01	-0.06	5.35	1.79	1.69	9.67
	40	3.96	2.20	13.28	0.77	-0.10	13.08	-3.27	-3.53	20.24
5	80	1.56	0.93	8.16	0.27	-0.07	7.58	-1.23	-1.47	13.46
	160	0.62	0.34	5.50	0.05	-0.06	4.89	-0.47	-0.61	9.40

Table A3. Bias (Mean and Median) and Efficiency (SD) for four observations per inter-quantile interval

		Mu			Sigma			Tau		
K	n	Mean	Median	SD	Mean	Median	SD	Mean	Median	SD
1/3	40	-12.37	-8.00	31.90	-9.15	-6.18	20.52	13.78	6.51	30.05
	80	-6.86	-3.65	24.29	-4.85	-3.35	12.88	7.39	3.23	22.45
	160	-3.64	-2.48	18.54	-2.78	-1.94	8.67	3.78	2.74	17.21
1/2	40	-2.70	1.01	32.44	-6.51	-3.41	21.04	3.82	-2.33	31.40
	80	0.48	2.25	25.32	-2.74	-1.10	13.64	-0.12	-2.32	24.11
	160	2.58	2.40	19.36	-0.68	0.14	9.31	-2.49	-2.17	18.46
1	40	7.63	6.08	31.80	-1.76	0.00	22.11	-6.96	-6.04	33.90
	80	4.61	2.04	23.49	-0.06	0.05	14.31	-4.46	-1.63	24.49
	160	2.29	0.86	15.69	0.20	0.00	9.57	-2.23	-0.76	16.01
2	40	5.91	2.73	23.71	-0.75	-0.90	20.18	-5.34	-2.94	28.76
	80	2.01	0.90	14.77	-0.08	-0.24	12.09	-1.83	-1.22	17.83
	160	0.62	0.28	9.62	-0.18	-0.29	7.55	-0.50	-0.35	11.60
3	40	5.40	2.83	18.44	-0.40	-0.98	18.38	-4.93	-3.91	24.73
	80	1.55	0.84	11.35	-0.31	-0.40	10.71	-1.34	-1.29	15.57
	160	0.63	0.36	7.49	-0.06	-0.14	6.43	-0.50	-0.57	10.42
4	40	5.32	3.06	15.98	0.26	-0.82	17.04	-2.68	-2.31	22.82
	80	1.60	0.85	9.66	-0.26	-0.29	9.90	0.69	0.61	14.56
	160	0.61	0.36	6.28	-0.06	-0.13	5.79	1.71	1.60	9.83
5	40	5.49	3.22	14.51	1.09	-0.53	16.26	-4.88	-4.71	21.86
	80	1.76	1.03	8.43	-0.27	-0.31	9.34	-1.72	-2.01	13.79
	160	0.65	0.36	5.59	-0.03	-0.12	5.35	-0.56	-0.69	9.53

Table A4. Bias (Mean and Median) and Efficiency (SD) for eight observations per inter-quantile interval.

		Mu			Sigma			Tau		
K	n	Mean	Median	SD	Mean	Median	SD	Mean	Median	SD
1/3	80	-9.94	-6.83	25.21	-6.30	-4.15	14.64	11.03	6.80	24.43
	160	-5.18	-3.43	18.91	-3.35	-2.34	9.28	5.47	3.59	17.88
1/2	80	-1.55	0.77	26.10	-3.98	-1.58	15.48	2.36	-1.12	25.84
	160	1.84	2.33	19.84	-1.08	0.02	9.92	-1.67	-2.26	19.25
1	80	5.85	3.52	25.38	-0.29	0.58	16.48	-5.67	-3.70	27.33
	160	2.77	1.04	16.91	0.21	0.08	10.38	-2.76	-0.98	17.66
2	80	2.99	1.19	16.48	-0.39	-0.37	14.93	-2.93	-1.67	20.42
	160	0.77	0.32	9.95	-0.23	-0.32	8.28	-0.68	-0.46	12.16
3	80	2.35	1.19	12.30	-0.75	-0.61	13.69	-2.17	-1.65	17.16
	160	0.71	0.44	7.74	-0.21	-0.20	7.31	-0.63	-0.67	10.84
4	80	2.44	1.34	10.34	-0.75	-0.56	12.95	-0.19	0.01	15.71
	160	0.68	0.43	6.49	-0.21	-0.16	6.84	1.56	1.53	10.17
5	80	2.73	1.69	9.10	-0.59	-0.52	12.30	-2.69	-2.64	14.90
	160	0.72	0.38	5.74	-0.32	-0.20	6.63	-0.74	-0.88	9.77

Table A5. Bias (Mean and Median) and Efficiency (SD) for sixteen observations per inter-quantile interval

		Mu			Sigma			Tau		
K	n	Mean	Median	SD	Mean	Median	SD	Mean	Median	SD
1/3	160	-7.92	-6.34	19.46	-4.37	-3.04	10.29	8.61	6.65	19.00
1/2	160	0.33	1.42	20.31	-1.79	-0.30	10.88	0.11	-1.26	20.29
1	160	3.91	2.06	19.04	0.28	0.50	12.01	-3.92	-2.35	20.64
2	160	1.14	0.47	10.92	-0.39	-0.37	10.17	-1.09	-0.62	13.81
3	160	1.01	0.53	8.27	-0.62	-0.38	9.69	-0.92	-0.78	11.84
4	160	0.99	0.55	6.87	-0.92	-0.45	9.51	1.26	1.36	10.94
5	160	1.15	0.66	6.03	-1.11	-0.49	9.34	-1.20	-1.18	10.37