Running Head: Evaluating Perceptual Integration

Evaluating Perceptual Integration: Uniting Response Time and Accuracy Based Methodologies

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Abstract

This investigation brings together response-time, system identification methodology (e.g., Townsend & Wenger, 2004a) along with accuracy methodology intended to assess models of integration across stimulus dimensions (features, modalities, etc.) proposed by Shaw and colleagues (e.g., Mulligan & Shaw, 1980). The goal was to theoretically examine these separate strategies and to apply them conjointly to the same set of participants. The empirical phases were carried out within an extension of an established experimental design called the *double factorial paradigm* (e.g., Townsend & Nozawa, 1995). That paradigm, based on response times, permits assessment of architecture (parallel vs. serial processing), stopping rule (exhaustive vs. minimum time) and workload capacity, all within the same blocks of trials. The paradigm introduced by Shaw and colleagues uses a statistic formally analogous to that of the double factorial paradigm but on accuracy rather than response times. We demonstrate that the accuracy measure cannot discriminate between parallel and serial processing. Nonetheless, the class of models supported by the accuracy data possess a suitable interpretation within the same set of models supported by the response time data. The supported model, consistent across individuals, is parallel, limited capacity, with the participants employing the appropriate stopping rule for the experimental setting. [200 words]

Key words: Response Time, Accuracy, Parallel Processing, Redundant Targets, Interaction Contrast, No Probability Response Contrast, Integration, Coactivation, OR task, AND task

How does the cognitive system combine information from separate sources? This question is central to basic human information-processing and also possesses many potential applications from clinical science to human factors and engineering. In the current study, we bring together two previously distinct approaches that can combine to provide strong converging evidence about some of the critical properties of human information-processing. The approaches are applicable to the two primary measures of performance in psychological research, response accuracy and response times (hereafter, RTs), so considered together they allow strong inference regarding the mechanisms underlying cognitive performance.

With regard to the measure of RT, we employ Townsend and Nozawa's (1995) systems factorial technology (hereafter SFT) framework, and expand it empirically as we outline shortly. With regard to the measure of response accuracy we build on the seminal efforts of Marilyn Shaw and colleagues (e.g., Mulligan & Shaw, 1980; Shaw, 1982). Her work, in her own terminology, was oriented toward the issue of perceptual integration vs. separate processing of multiple inputs. A key ingredient in her approach was the *"NO" response probability contrast* (NRPC), which we also define soon. The NRPC statistic permitted Shaw and colleagues to disconfirm several classes of models and provide support for one (Mulligan & Shaw, 1980; Shaw, 1982) for the stimuli they considered.

However, several major questions remained unanswered. Since the experiments used to test their models are all accuracy based, the models are not given time-oriented, dynamic explanation. For instance, Shaw's response-accuracy measure cannot tell whether processing is parallel or serial, or what is the system's ability to handle changes in processing workload over time. Thus, we seek to enlist both response time and accuracy measures in order to more completely understand underlying properties of perceptual systems. An important advantage of the basic measures we consider in this study (both RT and accuracy measures), is that they are non-parametric and are thus robust across many specific parameterized models.

Another limitation of Shaw's approach was that it was applicable only to tasks with an OR decision rule. Here we develop a new, appropriate contrast for AND tasks, and derive the relevant model predictions. To illustrate the difference between OR and AND rules used to combine the information prior to response, consider a display with two (or more) signals. In the OR case, participants may be asked to detect the presence of one signal *or* another *or* both. In the AND case, response is required only if signals are presented on both channels (e.g., channel A *and* B). Our study offers a complete investigation of human performance with OR and AND rules, in terms of both RT and accuracy, without making specific parametric assumptions.[1]

We begin by outlining several important properties of the human information-processing system, and the RT- and accuracy-based tools used to identify these properties. Our exposition of the experiments is then divided into a response-time section (Study I) and an accuracy section (Study II) and within each to OR and AND experiments. We first present a new RT study (Experiment 1 -- OR) employing the double factorial design and the systems factorial methodology devised for it (Townsend & Nozawa, 1995; see explication below). We further present a second new RT experiment with a different logical processing rule (Experiment 2 -- AND). In Study II we report a third new experiment with the Shaw-type paradigm and analyses involving accuracy (Experiment 3 -- OR), and extend them by adding an experiment based on a different logical processing rule (Experiment 4 -- AND), analogous to the novel design in our RT paradigm (Experiment 2). Experiments 2 and 4 are, to the best of our knowledge, the first empirical (perceptual) tests for these factorial techniques in AND designs. Moreover, to analyze

[1] Models of speeded decisions, such as the diffusion model (Ratcliff, 1978) or the linear ballistic accumulator (Brown & Heathcote, 2008) successfully account for choice latency and accuracy. However, they make very specific assumptions about the form of the distributions and their parameters, which we are able to avoid here.

the AND accuracy data we extended Shaw's machinery and derived an appropriate accuracy measure for AND designs.

Following a discussion of the two sets of experiments, a unified theoretical framework is presented which permits placing the RT and accuracy based approaches within a common framework. In beginning, our general approach is outlined. We will then be in a position to interpret Shaw's models within that extended theory and methodology.

Basic Properties of the Human Information-Processing System

When presented with signals from multiple sources, say, from two different spatial locations, there is a number of aspects of information processing that are basic in characterizing the perceptual or cognitive systems. First, within the issue of *architecture*, people may process both signals at the same time that is, in *parallel*, or process one first and then process the other, that is, *serial* processing. Second, the cognitive system may employ different *stopping rules*: it can complete the processing of both signals, also called the *exhaustive* stopping rule, or it can halt processing after the completion of only one signal, the *minimum-time* or *first-terminating* stopping rule. Third, *workload capacity,* denoted C(t), refers to processing efficiency as a function of workload (e.g., Townsend & Ashby, 1983; Townsend & Wenger, 2004b). Specifically, C(t) measures the relative cost vs. benefit to performance when an additional channel or information source is added in the stimulus. Finally, *independence* (or the lack of) refers to possible *interdependencies* between different processing channels.

Systems factorial technology comprises a set of possible approaches to identifying the above system properties within a unified framework (Townsend, 1992; Townsend & Wenger, 2004a). This approach entails an interrelated taxonomy for elementary cognitive processes (Townsend, 1974; Townsend & Ashby, 1983), augmented by a mathematical theory and associated experimental methodology (Townsend & Nozawa, 1995; Townsend & Wenger, 2004a) to experimentally characterize the psychological system of interest. The systems factorial methodology employs response times for assessing the different dimensions of processing.

The *redundant-target task* had been proven useful in studying the above issues. In one version of such a task, participants may be presented with a target signal on one location, on another, or on both (there also exists a non-target display, which, depending on the particular procedure, can be either blank or comprised of non-target items). Participants are instructed to respond affirmatively if they detect at least one target, i.e., if a target appears in one location, *or* the other, *or* both (a disjunctive rule), hence the name OR task. The condition where two targets appear is called a redundant target (sometimes double target) condition, since one target is sufficient for the participant to respond affirmatively.

In a different version, the same stimulus-types as in the OR design may appear but now the instructions are to respond “YES” if and only if *both* locations are occupied by targets, i.e., if there is a target in one location *and* in the other (a conjunctive rule) and therefore the name AND task. In each basic design, in principle, participants might extract information from the two spatial locations serially, in parallel, or in some hybrid fashion. The choice of stopping rule, however, should be dictated by the task demands, if participants are to conform to the instructions and perform accurately with most efficiency. Thus, in the OR design while a participant might still process both targets on a redundant trial, due to choice or inability to do otherwise, such an option is not as efficient as stopping as soon as the first is completed. Conversely, in an AND task, the task imposes an exhaustive stopping rule.[2]

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[2] There is an inherent duality between “YES” vs. “NO responses that logically interacts with the stopping rule. With an OR design, to correctly respond “NO” on no-target trials, participants must confirm that a signal is missing from channel A and B, hence there is virtually an AND stopping rule with regard to the “NO” decision. Conversely, in an AND design (to respond “YES”, the participant must make sure there is a signal in each channel), a “NO” decision

Except for serial and parallel modes of processing, another important type of architecture is again parallel but rather than each channel handling its own detection, it is assumed that the information or activation within each channel is added together with that from the other channel in a subsequent pooled outlet. This final pooling channel possesses a detector-criterion for deciding if, at any point in time, there is sufficient support to report the presence of a signal, from either or both input channels. In the redundancy literature, this type of system is typically referred to as a *coactive* system (e.g., Colonius & Townsend, 1997; Diederich & Colonius, 1991; Miller, 1982; Schwarz, 1994; Townsend & Nozawa, 1995; Houpt & Townsend, 2011). For a "NO" response to occur, it must be the case that the added activations fail to meet the criterion. The logical notion of a stopping rule becomes vacuous in a coactive system, because the decision threshold is only assessing activation on the single 'final' channel.[3] A coactive system with a single, common detection mechanism that aggregates activation from all channels before decision is illustrated in Figure 1b, and can be compared with a separate-channel parallel system in Figure 1a.

becomes in effect, an OR trial, since if either channel delivers a "no-signal" decision, the overall decision can be "NO" and processing can cease with the first "NO" decision to occur. Importantly, these response strategies are testable with our tools.

[3] Thus, a coactive system cannot, in a logical sense, perform certain versions of AND tasks because it cannot make separate decisions on the different channels. Nonetheless, a coactive model could, in principle, simply increase its decision criterion so as to attempt to minimize mistakes when only one or no signal is present.

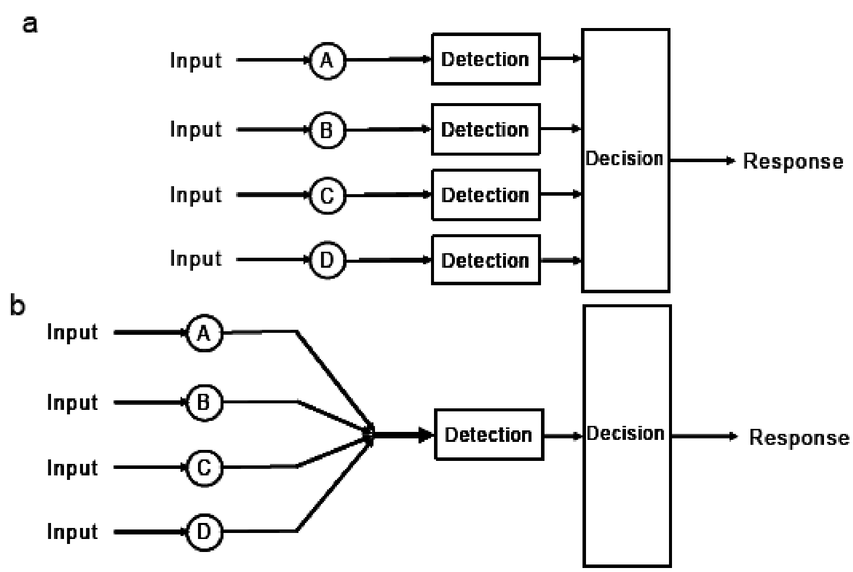

Figure 1. A schematic illustration of parallel-independent (a) and coactive (b) architectures.

Theoretical and Empirical Developments Brought by the Current Investigation

The current investigation pursues the issues of architecture, stopping rule, capacity, and independence. It involves both OR as well as the AND designs and formulates an RT study as well as an accuracy study, the latter following along the lines of Shaw and colleagues. Within the RT analyses, we can discern architecture, capacity, and stopping rule. Independence is indirectly and partially assessable within the RT paradigm (see section Workload Capacity). In contrast, independence appears as a major component in the predictions in the accuracy analyses and the stopping rule provides robust implications as well when the accuracy-based NRPC is used. Architecture is less precisely assayed in the accuracy experiments. However, the combination of response time and accuracy together permit, under assumptions of parsimony, a unified account of both data sets.

Theoretical and empirical aspects are necessarily intertwined in this research, but segregation according to the major contributive features will aid the presentation. The current work offers a number of experimental contributions. Predominantly, the overall goals of this investigation employed a combination of response time and accuracy experimental paradigms for purposes of seeking a unified interpretation, comparison and linkage of RT and accuracy models. We show that the linkage of RT with accuracy permits converging evidence of architecture, stopping rules and performance efficiency not available with either alone.

Furthermore, to allow a direct comparison between performance on RT and accuracy tasks, it was important to use the same participants in all the conditions, which of course, has not been done before. Finally, it was also desirable to employ a very simple and reasonably well understood basic paradigm and stimuli to facilitate comparisons across the accuracy and RT designs. This was accomplished in such a way as to fulfill a need recognized by Shaw and colleagues, as detailed below. We used a simple dot detection task close to that of Townsend and Nozawa (1995). That experiment recorded response times in a redundant-target task with two dots, but used only the OR design, finding overwhelming evidence for parallel, minimum-time processing with capacity varying from super to quite limited. The present RT study contained also an AND condition which requires exhaustive (conjunctive) processing. We assess architecture and probe capacity with an alternate measure of capacity, appropriate for AND experiments. On that note, Shaw (1982) and Mulligan and Shaw (1980) had also focused only on OR decisions. Thus, our new AND accuracy experiment effectively extends both the theoretical and the empirical domain of the original papers.

The final experimental contribution of this paper involves adjustment to the procedure used by Mulligan and Shaw (1980), to potentially allow for integration of information. The data and analyses of Mulligan and Shaw supported independent decision models, not integration as they defined it in their terminology. Their stimuli, however, were presented peripherally (40 degrees off center). Miller (1982), Berryhill, Kverga, Webb, & Hughes (2007), and others, with response times and central presentations, have found evidence against race (separate channels) models and in favor of coactive models. It is possible that information is integrated differently on and off the center of the perceptual field. Therefore, in Mulligan and Shaw's words: "…replication with stimuli at 0 deg azimuth…would be a worthwhile endeavor." (p. 476). Of course, no two dots can occupy the same place in space at 0 deg azimuth, but our stimuli were fairly close to it (±1 deg above and below a central fixation point).

On the theoretical side, we provide a unified approach and taxonomy for the Shaw models within our theory and models. Furthermore, we extend Shaw's family of model predictions to AND paradigms. Finally, we prove that architecture and the appropriate temporal dynamics play virtually no role in the NRPC accuracy predictions. That is, different processing architectures (serial, parallel) predict the same NRPC pattern. However, both Shaw's models and our models make differential predictions concerning the OR and AND stopping rule, independent of architecture. That is, the stopping rule is critical but not the architecture. Our coactivation model and the Mulligan and Shaw (1982) weighted integration model are excluded here because as noted earlier, the concept of stopping rule is inapplicable so they make identical predictions for OR and AND designs.

In the upcoming section we shall briefly recount two tests for assessing the stopping rule and architecture that are based on response time distributions: mean interaction contrast, and survivor interaction contrast. We will further outline a third measure, the workload capacity coefficient, which assesses the processing capacity of the system, as workload varies, and at the same time indirectly informs us about architecture. We will then survey the methodology of Shaw and colleagues (Mulligan & Shaw, 1980; Shaw, 1982), in which response accuracy is the independent variable. Then we present data from two studies, each involving two experiments, in which the same participants performed with high-accuracy (response time task) and para-threshold stimuli (accuracy task). We now turn to a brief presentation of our theory-driven methodology.

Analysis of Response Time Data: System Factorial Technology

Systems factorial technology is a theory-driven experimental methodology that allows for a taxonomy of four critical characteristics of the cognitive system under study: architecture (serial vs. parallel), stopping rule (exhaustive vs. minimum-time), workload capacity (limited, unlimited, or super) and channel independence. The first three are directly tested by our response time methodology. Independence can only be indirectly assessed, as opposed to accuracy, through measures such as capacity (e.g., Townsend & Wenger, 2004b). Architecture and stopping rule are the primary characteristics targeted in this study, but we shall see that capacity and possibly channel dependencies may be implicated in the interpretations.

Systems factorial technology analysis is based on a factorial manipulation of two factors with two levels, and it utilizes two main statistics: the mean interaction contrast (MIC; Ashby & Townsend, 1980; Schweickert, 1978; Schweickert & Townsend, 1989; Sternberg, 1969) and the survivor interaction contrast (SIC; Townsend & Nozawa, 1995). The latter extension makes use of data at the distributional level rather than means and therefore permits analysis at a more powerful and detailed level (Townsend, 1990; Townsend & Nozawa, 1988, 1995). Both statistics are independent of the underlying stochastic distribution. The only real assumption necessary to propel this methodology and to calculate MIC and SIC is that of *selective influence*. The concept of selective influence was treated as being equivalent to statistical main effects at the level of means for many years, in the sense that, for instance, a higher level of salience of a stimulus will lead to a significantly faster mean RT. It is now acknowledged that selective influence must act at the level of ordering the RT distributions, not just means (Townsend & Ashby, 1983; Townsend

& Schweickert, 1989; Townsend, 1990). Townsend, Dzhafarov, and their colleagues (Dzhafarov, 2003; Kujala & Dzhafarov, 2008) continue to investigate the underlying theory and underpinning conditions for selective influence.

Mean Interaction Contrast

The MIC statistic describes the interaction between mean response times (MRT) of two factors with two levels each and can be presented as follows:

$$MIC = (MRT_{LL} - MRT_{LH}) - (MRT_{HL} - MRT_{HH}) = MRT_{LL} - MRT_{LH} - MRT_{HL} + MRT_{HH}.$$

There are two subscript letters; the first denotes the level of the first factor (H=high, L=low) and the second indicates the level of the second factor. For the sake of concreteness, consider for example the visual target-detection task that we use in Experiment 1. The two factors in this task may be the salience (contrast, intensity) levels of each of two bright dots displayed against dark background. The first factor may then be the salience of a target presented on the top position and the second factor may be the salience of a target presented at the bottom. Thus, the first and second subscript letters refer to the intensity level (H, L) of the top and bottom targets, respectively. Note that MIC gives the difference between differences of mean response times, which is literally the definition of interaction. MIC=0 indicates that the effect of one factor on processing latency is exactly the same, whether the level of the other factor is L or H. Conversely, if two factors interact, then manipulating the salience of one factor would yield different effects depending on the level of the other factor, hence $MIC \neq 0$. Under-additive interaction, or $MIC<0$, is a typical prediction of parallel exhaustive processing, while over-additivity, or $MIC>0$, is associated with parallel minimum-time processing or coactive models. Serial models, with either an exhaustive or a minimum-time stopping rule, predict additivity, or MIC=0 (Townsend & Ashby, 1983; Townsend & Nozawa, 1995).

Survivor Interaction Contrast

The survivor interaction contrast function (SIC) is defined as:

$SIC(t) = [S_{LL}(t) – S_{LH}(t)] - [S_{HL}(t) – S_{HH}(t)]$,

where *S(t)* denotes the response time survivor function. In brief, to calculate the SIC we divide the time scale into bins (say, of 10 ms each) and calculate the proportion of responses given within each time bin to produce an approximation to the density function, *f(t)*, and the cumulative probability function, *F(t)*. That is, *F(t)* is equal to the probability that response time is less than or equal to *t*. The survivor function, *S(t)*, is the complement of the cumulative probability function [1- *F(t)*] = *S(t)* and tells us the probability that the process under study finishes later than time *t*. To produce the SIC, one calculates the difference between differences (hence, the interaction term, as the name suggests) of the survivor functions of the four corresponding factorial conditions the way it is derived for the means, but does so for every bin of time.

Note that this statistic produces an entire function across the values of observed response times. Furthermore, there is a specific signature of each architecture and stopping rule, with respect to the shape of the SIC function (Townsend & Nozawa, 1988, 1995). For example, the SIC function for a parallel minimum-time model is positive for all time *t*, whereas the SIC function of a coactive model starts negative and then crosses the abscissa and becomes positive. If we integrate the SIC function from zero to infinity it is known to give the MIC (Townsend & Nozawa, 1995), and in both models the MIC actually turns out to be positive. So, it is the finer grained SIC that allows for a decisive test between a coactive and a parallel minimum-time mode of processing. Although the MIC is not nearly as diagnostic as the SIC, it reinforces the SIC results and provides a means of statistically assessing any interactions associated with the SIC function. The MIC and SIC predictions of parallel, serial, and coactive models are summarized on the right hand side of Table 1.

Workload Capacity

By “workload capacity” we refer to the processing efficiency of the system as we increase the load of information by increasing the number of the to-be-processed targets. Townsend and Nozawa (1995) proposed a measure of performance under increases in workload that is based on hazard functions. The hazard function, $h(t) = f(t)/S(t)$, captures the likelihood that a specific process (e.g., channel) will finish processing in the next instant, given that it is not yet done. The larger the hazard function at any point in time, the higher the speed of processing. The integrated hazard function, $H(t)$, is the integral of the hazard function from zero to t. The associated statistic is the *capacity coefficient*, which is computed as the ratio of the integrated hazard function from the double-target condition (i.e., two targets presented simultaneously viewed here as ‘AB’) and the sum of the integrated hazard functions of the single-target conditions A, B respectively: Thus, $C_{OR}(t) = H_{AB}(t)/[H_A(t)+H_B(t)]$. The subscripts OR indicate that this index is calculated for the OR task.

$C_{OR}(t)$ is a measure benchmarked against a standard parallel process, with stochastically independent channels. The benchmark model is unlimited capacity in the sense that its channel speeds do not vary with the number of other channels that are operating. Any such models produce $C_{OR}(t) = 1$ for all times $t \geq 0$. The prediction of a standard serial model, where processing of one target item has to be completed before commencing the processing of the other target item and the two processes are independent, is, under certain conditions $C_{OR}(t)=.5$.

It important to note, however, that distinct architectures might produce the same capacity values. For example, a parallel model with inhibition across processing channels can produce C(t) values close to .5 , which are characteristic of a serial system (Eidels, Houpt, Altieri, Pei, & Townsend, 2011). Such potential model mimicry emphasizes the importance of employing tests such as MIC and SIC that assess architecture while avoiding conflation with capacity.

To sum up, $C_{OR}(t)$ values of 1 imply that the system has an unlimited capacity. $C_{OR}(t)$ values that are below 1 define limited capacity, such that increasing the processing load (by increasing the number of targets on the display) takes toll on the performance of one or both channels. Finally, if $C_{OR}(t) > 1$ then the system is said to have super-capacity; processing efficiency of individual channels actually increases as we increase the workload. Although capacity and independence are logically distinct, the capacity coefficient can be affected by dependencies. Thus, the prediction of a parallel model with positive cross-channel interactions is $C_{OR}(t) > 1$, as is the qualitative prediction of a coactive model (Eidels et al., 2011). Very strong inhibitory cross-channel interactions, may lead to severely limited capacity, such that $C_{OR}(t) < .5$.

Townsend and Wenger (2004b) developed a comparable capacity index for the AND task: $C_{AND}(t) = [K_A(t)+K_B(t)]/ K_{AB}(t)$. Here, K(t) is the integral of a different kind of 'hazard' function, one that calculates the likelihood of just finishing in the last instant, rather than the instant ahead, and conditioned on the event that the process has been completed until just around time t: $k(t)=f(t)/F(t)$, and $K(t) = \int k(t') dt'$, integrated from zero to t. In $C_{AND}(t)$, the numerator and denominator are arranged such that the interpretation is identical to that of the OR capacity index: $C_{AND}(t)$ values that are above, at, or below 1 imply super-, unlimited-, or limited-capacity, respectively.

Miller (1978, 1982) suggested an upper bound on response-time distribution when independent channels are involved in a race (in an OR task). He suggested that a violation of this bound, colloquially known as Miller's race-model inequality, is evidence against race models and in favor of coactivation.[4] Townsend and Eidels (2011) showed that it could in fact be viewed as a measure of workload capacity rather than architecture, and that it can be mapped onto the same

[4] A coactive model naturally predicts that responses on double-target displays would be faster than on single-target displays (the redundant target effect). Raab (1962) noted that a stochastic independent-channels race model can also produce this effect via statistical considerations alone. Miller's inequality helps deciding between the two models.

space as C(t). Since Miller's race-model inequality is a conservative test (a model can have super capacity and not violate the inequality), we use instead the more refined C(t) measure.

Analysis of Response Accuracy Data: The "NO" Response Probability Contrast (NRPC) and Mulligan & Shaw's Information-Sampling Models

No Response Probability Contrast

We next consider the application of a factorial method to a psychophysical experiment in which response accuracy is the dependent variable. As noted earlier, the present development represents an extension of the work of Shaw and colleagues (e.g., Shaw, 1982; Mulligan & Shaw, 1980) within the confines of our systems factorial oriented approach. Mulligan and Shaw (1980) analyzed the probabilities of "NO" responses in each of four stimulus conditions using the following formula, which we call the "NO" response probability contrast (NRPC) as related earlier. Let Ø represent a blank or null stimulus, A the presence of a target at the top of the display, and B the presence of a target at the bottom of the display. Then we define the "NO" response probability contrast as the double difference,

NRPC = P[NO|(Ø,Ø)] – P[NO|(A,Ø)] – (P[NO|(Ø,B)] – P[NO|(A,B)]).

The first term, P[NO|(Ø,Ø)], represents the probability of a "NO" response given no signal. The term P[NO|(A,B)] represent the probability of a "NO" response given signals on both the top and bottom positions (i.e., a double target display), and so on.

Mulligan and Shaw (1980) derived predictions for the probabilities of "NO" responses (as well as for their logarithmic and z score transformations) for four types of models. The predictions of these models are summarized on the left hand side of Table 1. The description of these models follows shortly, but for the readers' convenience we also summarize the formulas in Table 2.

We shall employ Mulligan and Shaw's terminology to facilitate connections with their earlier papers, and subsequently place the models within our approach. It should be noted that on occasion the language in their papers may seem to suggest a wider interpretation of the models than what was mathematically defined in their equations. We must confine our analyses to the published mathematical interpretations.

All four models assume that, much like the static theory of signal detectability (e.g., Green & Swets, 1966), information is compared with one or more criteria. The first three models postulate that a stochastically independent comparison is made on each channel of the sampled information vs. a criterion. These three models are said to differ from one another in the way attention is allocated to multiple processing channels (say, to visual and auditory modalities, or in our case display positions). In the fourth model, information from the two channels is averaged prior to comparison, and this single integrated value is then compared to a single criterion. Next, we examine more closely each of the models and its NRPC predictions.

Independent-decision sharing model. In this model (also dubbed the fixed-sharing model in Shaw's 1982 paper) the participant is viewed as sharing attention between channels on each trial, and the proportion of attention assigned to each channel is assumed to remain constant across trials. The formula for the probability of a NO response is $P("NO") = P(X_A < \beta_A) \cdot P(X_B < \beta_B)$. If each processing channel accumulates counts until a prescribed number is reached, then X_A and X_B are the random variables that represent the number of counts on processing channels A and B, and β_A and β_B are the respective decision criteria.

Mulligan and Shaw (1980) showed that this model predicts an over additive NRPC (i.e., NRPC > 0) and additivity after logarithmic transformation, such that log(P[NO|(Ø,Ø)] – log(P[NO|(A,Ø)]) – log(P[NO|(Ø,B)]) + log(P[NO|(A,B)]) = 0 (see also Table 1 -- 'Sharing' model). Our independent, unlimited capacity parallel (race) models, when adapted to accuracy designs, can make this type of prediction. (e.g., Townsend & Ashby, 1983, Chapter 9). Now, within our approach ‘sharing’ usually implies limited capacity since it seems to suggest a fixed or bounded source of capacity in which $H_{AB}(t)$ would be less than $H_A(t) + H_B(t)$ (see section Workload Capacity above). However, since we confine our discussion to the mathematical expression of the model, as presented by Mulligan and Shaw, it should be viewed for all intents and purposes as having unlimited capacity.

Independent-decision all-or-none probability mixture model. This model is a probability average of the probability that in each of the two channels information fails to reach its criterion. The formula relating individual channels to the overall likelihood of a NO response is $P("NO") = \alpha \cdot P(X_A < \beta_A) + (1-\alpha) \cdot P(X_B < \beta_B)$. Information on each trial is obtained from only one channel: only from channel A, with probability α, or only from channel B with probability (1-α). The overall performance is then a weighted mixture of performance on individual trials. This model can be viewed as an attention-switching model where on any single trial attention is fully allocated to one channel and not the other, but can switch between channels on subsequent trials. Of course, the unattended source of information, or channel, has no influence on response probabilities.

The prediction of this version of the mixture model, also termed the all-or-none mixture model (Shaw, 1982) is NRPC = 0 (Table 1 -- ‘Mixture: all or none’). Within our taxonomy, this type of model would be classified as a serial model which, with some probability selects one of the channels to process and stops immediately after completion (although, as we show in the General Discussion, all four models in Table 1 can be viewed as either serial or parallel). This type of behavior is most appropriate when responding “YES” on redundant target trials -- this is called first-terminating (or minimum time) processing. Of course, a "NO" response requires rejection on both trials and hence demands exhaustive processing to have a chance for optimal performance. We would expect accuracy with this kind of model to be sub-optimal.[5]

Independent decision not all-or-none mixture mode. In this version of a mixture model, attention is directed primarily to one source, but some information from the unattended source is used in the detection decision. Attention affects the criterion value, such that β is the criterion for the attended channel and β' is the criterion for the less (but still) attended channel. Its formula is $P(\text{"}NO\text{"}) = \alpha \cdot P(X_A < \beta_A) \cdot P(X_B < \beta'_B) + (1-\alpha) \cdot P(X_A < \beta'_A) \cdot P(X_B < \beta_B)$. The prediction of this version is NRPC > 0 (Table 2 -- 'Mixture: not all or none'). This type of model would be called a compound processing model in our approach (see Townsend & Ashby, 1983, Chapter 5) since this kind of prediction would follow from a probability mixture of parallel systems.

Weighted integration model. In this model, averaged evidence from separate processing channels is summed prior to decision and then compared to a single criterion, β. Its formula is $P(\text{"}NO\text{"}) = P([w \cdot X_A + (1-w) \cdot X_B] < \beta)$. Note that this formula is compatible with a system with attentional weight "w" placed on channel A and "1-w" on channel B. This model is a relative of the coactive model discussed earlier except that the convention for coactive models has become a simple addition of the information or activation from the separate channels (e.g., Colonius & Townsend, 1997). In addition, in the particular instantiation studied by Mulligan and Shaw (1980) the probability distributions of the internal random variables (evidence or activation

[5] Consider the OR task, where detection of at least one target signal is sufficient to elicit a "YES" response. If, on any given trial, attention in an all-or-none mixture model is fully allocated to one channel but not to the other, then at any time t there is just one process going on, much like in a serial model. On double target trials the system halts after processing the attended signal, no matter whether attention is allocated to one channel or the other (both have target signals). This case is identical to serial processing with a first-terminating stopping rule. Notice that both models predict very high error rate on single-target displays (50% if the probability of processing one channel but not the other or vice versa are equal): a serial first-terminating model that processes the target-absent channel first will stop before processing the second, target present display, leading to an erroneous "NO" response. Similarly, an all-or-none mixture model that allocates attention only to the target-absent channel will overlook the target presented to the other channel, again leading to an erroneous "NO" response.

in a channel) are assumed to be Gaussian. With an additional and rather strong assumption of equal variances of the signal and the no-signal distributions, the weighted integration model predicts additivity of the z, or inverse-normal transformations of the probabilities of a "NO" response: z(P[NO|(Ø,Ø)]) – z(P[NO|(T,Ø)]) – z(P[NO|(Ø,B)]) + z(P[NO|(T,B)]) = 0 (Shaw, 1982, pp. 373-376).

Table 1. *Different processing models and their accuracy and response time predictions.*

Mulligan and Shaw (1980) and our time-based accuracy models' predictions		Response time models' (e.g., Townsend & Nozawa, 1995) predictions	
Model type	Predictions	Model type	Predictions
Sharing OR AND	 NRPC>0; log NRPC=0 NRPC<0; log YRPC=0	Parallel-independent OR AND	 MIC>0; SIC(t)>0 MIC<0; SIC(t)<0
Mixture: all or none OR AND	 NRPC=0 ?	Serial OR AND	 MIC=0; SIC(t)=0 MIC=0; SIC(t)<0 for small t, >0 for large t.
Mixture: not all or none OR AND	 NRPC>0 ?		
Weighted integration	z NRPC=0	Coactivation	MIC>0; SIC(t)<0 for small t, >0 for large t.

NRPC = P[NO|(Ø,Ø)] – P[NO|(T,Ø)] – P[NO|(Ø,B)] + P[NO|(T,B)]
log NRPC = log(P[NO|(Ø,Ø)]) – log(P[NO|(T,Ø)]) – log(P[NO|(Ø,B)]) + log(P[NO|(T,B)])
log YRPC = log(P[YES|(Ø,Ø)]) – log(P[YES|(T,Ø)]) – log(P[YES|(Ø,B)]) + log(P[YES|(T,B)])
zNRPC = zscore(P[NO|(Ø,Ø)])–zscore(P[NO|(T,Ø)])–zscore(P[NO|(Ø,B)])+zscore(P[NO|(T,B)])

Table 2. *Formal description of four types of processing models that were studied by Mulligan and Shaw (1980). The equation for each model gives P("NO"), the probability of responding "no signal." For all models,* X_A *and* X_B *are the random variables that represent the number of counts on two processing channels, A and B, and* β_A *and* β_B *are the respective decision criteria. The last model has only one criterion,* β*. See text for clarification concerning other notation.*

Model	Equation
Independent-decision sharing model	$P(\text{"}NO\text{"}) = P(X_A < \beta_A) \cdot P(X_B < \beta_B)$
Independent-decision mixture all-or-none model	$P(\text{"}NO\text{"}) = \alpha \cdot P(X_A < \beta_A) + (1-\alpha) \cdot P(X_B < \beta_B)$
Independent-decision mixture not all-or-none model	$P(\text{"}NO\text{"}) = \alpha \cdot P(X_A < \beta_A) \cdot P(X_B < \beta'_B) + (1-\alpha) \cdot P(X_A < \beta'_A) \cdot P(X_B < \beta_B)$
Weighted integration model	$P(\text{"}NO\text{"}) = P([w \cdot X_A + (1-w) \cdot X_B] < \beta)$

To recap, the independent sharing model and the not-all-or-none mixture model both predict NRPC > 0. The former also predicts the double difference of the log transforms of the probabilities of the NO responses will equal 0. The model with an all-or-none mixture predicts NRPC = 0. Finally, the weighted integration model, a relative of the coactive model, predicts z NRPC = 0. These predictions are summarized, as we mentioned, on the left hand side of Table 1.

We next present the two experiments of Study I, which employed RTs to directly assess architecture, stopping rule, and workload capacity. We then proceed to study II (Experiments 3 and 4), where the same individuals performed in accuracy tasks.

Study I: The OR and AND Response Time Experiments

Method

Participants. Ten Indiana University students (two graduates and eight undergraduates; 7 females, 3 males) were paid to participate in the study. They had normal or corrected to normal vision. Their ages ranged between 22 and 30 years. The participants performed in 8 experimental sessions of approximately an hour each.

Stimuli. There were 9 possible stimulus displays: 4 types of double target displays, 4 types of single target displays, and 1 no-target display. On a double target display two dots, with a diameter of .2° each, were located on a vertical meridian, equally spaced above and below a fixation point at an elevation of ±1°. We refer to these targets as the top (A) and bottom (B) signals respectively. There were two levels of target luminance (67 cd/m^2 and .067 cd/m^2), chosen after pilot testing to ensure a robust effect on the response times (to allow for testing the interaction contrasts). Each target could appear in high (H) or low (L) luminance, thus comprising a total of four possible combinations (HH, HL, LH, and LL). On a single target display, a target appeared at the top or at the bottom position, but not on both. The single dot could appear in high or low luminance. The target-absent display consisted of a blank black screen.

The stimuli were generated via Microsoft Painter by an IBM compatible (Pentium 4) microcomputer and displayed binocularly on a super-VGA 15’’ color monitor with a 1024x768 resolution using DMDX software (Forster & Forster, 2003). On a trial, a single-pixel fixation point (subtending to .05° of visual angle at a viewing distance of 50 cm; luminance of .067 cd/m^2) was presented on the center of the screen for 500 ms, followed by a blank black screen (500 ms), and then followed by the stimulus display. The stimulus appeared on the screen for 100 ms or until a response was given and was then replaced by a blank screen. Participants were instructed to respond as quickly as possible. The response sampling began with the onset of the stimulus display and continued for 4000 ms. The inter-trial-interval was 1000 ms. Participants were asked to respond affirmatively by pressing the right mouse key with their right index finger, and respond “NO” by pressing the left mouse key with their left index finger.

The probabilities of presenting both targets, presenting the top target alone, the bottom target alone, or no target at all were equal to .25.[6] The probabilities associated with each target luminance were .5.

Procedure. The participants were tested individually in a completely dark room, after 10 minutes of darkness adaptation. Each participant performed in both the OR and the AND experiments. In the OR experiment, participants were asked to respond affirmatively if they detected the presence of at least one target (i.e., two targets, single target on top, single target at the bottom), and respond “NO” otherwise. In the AND experiment, participants were asked to respond affirmatively if and only if they detected the presence of two targets, and respond “NO” otherwise. The order of experiments was counter balanced between participants. Each experiment consisted of four sessions, each about an hour long. Sessions were held on consecutive days (excluding weekend days). Feedback on response accuracy was given at the end of each session. Each session started with a practice block of 100 trials, followed by 5 experimental blocks of 160

[6] This means that the overall probability of a “YES” response in the OR task is .75, or .25 in the AND task, and a response bias may ensue. However, systems factorial technology is insensitive to such biases. First, it does not attempt to fit criterion parameters. More importantly, the different statistics [MIC, SIC(t), C(t)] are all based on trials from the same response (“YES”), so response bias makes no difference. The only exception might have been the $C_{AND}(t)$, but as we explain later in the text it is computed with single target data from the OR case (“YES” data).

trials each (with a 2 minute breaks in between blocks). The order of trials was randomized within a block. Overall, a large number of trials, 3200, were collected for each participant at each experiment (OR, AND), for tests at the distributional level.

Results and Discussion

One participant failed to reach the accuracy criterion (90%) and her data was therefore excluded from the analysis. Accuracy for the nine remaining participants was high in both the OR and the AND experiments. The overall error rate, across tasks and participants, was 3.6%, and no response time-error tradeoff was observed. Analyses of the response time data were restricted to correct responses in both experiments. Responses above 900 ms or below 160 ms were omitted from the analysis (based on pilot testing to approximate criteria of ±2.5 std. away from the mean). Because the primary interest in Experiment 1 and 2 was in the patterns of response time, we do not refer to accuracy in this section. Finally, because different individuals may employ different processing architectures (e.g., serial, parallel), or have different capacity limitations, we analyze and report separately results of individual participants. Group results (means) are presented at the bottom of Tables 3 through 6 to provide an overview, but were not subjected to separate inferential analysis.

Experiment 1 (OR): Mean reaction times for individual participants are presented in Table 3. For data pooled across participants (as well as for each of the individual participants), mean response times were fastest on the double target trials (314 ms), then next fastest on single target trials (342 and 353 ms for the single target top and bottom, respectively), and slowest on the target-absent trials (491 ms). This order was found to hold also at the survivor functions level, which implies a stronger level of stochastic dominance (cf. Townsend, 1990). These results are compatible with those of Townsend & Nozawa (1995).

We performed a 2x2 analysis of variance on the response time data of individual participants with the presence vs. absence of the top target as one factor, and the presence vs. absence of the bottom target as a second factor. The ANOVA revealed significant main effects at $p<.01$ for both factors and for all participants. That is, participants were faster to respond when a target was presented on the top position (328 ms when averaged across all participants) compared to trials with no target on top (422 ms). Similarly, they were faster to respond when a target was presented at the bottom position (333.5 ms across all participants) compared to trials with only a blank at the bottom (416.5 ms). For eight participants (all except BJ) there were also significant interactions of target-top x target-bottom, at $p<.001$, likely driven by the very slow responses on the no-target trials.

Comparing performance on double target vs. single target trials results in the capacity index, $C_{OR}(t)$. $C_{OR}(t)$ plots for individual participants are presented in Figure 2. Most $C_{OR}(t)$ coefficient values of each participant lie above .5 and below 1, suggesting moderately limited capacity throughout the processing interval. Note that values of $C_{OR}(t)$ above .5, given approximately equal performance on the two signals, also indicate a so-called race benefit, meaning performance is better than either target alone. The same pattern was observed for all individual observers. These results were qualitatively the same as the Townsend and Nozawa experimental condition where the double target stimuli presented the two dots dichoptically in corresponding retinal locations in the two eyes. It is also worth noting that reasonable base time components of RT (all the contributions to response times not involving the processes under inspection) will not lead to substantial distortion of capacity statistics. However, minor decreases in $C_{OR}(t)$ could be related to that variable (Townsend & Honey, 2007).

To better inform the reader about the (in)stability of the estimate of the capacity function, we plot in thin dashed lines the standard error of estimation (estimated by bootstrapping; see

Silverman, 1986, and Van Zandt, 2002). To overcome undesired effects of outliers we estimated $C_{OR}(t)$ for the time range containing 99% of the observations (separately for each individual). The estimations for this range were highly reliable, as evident by the exceptionally tight error bounds.

Table 3. *Mean response times (in ms.) in Experiment 1 (OR task).*

Participant	Double target	Single target top	Single target bottom	No target	Subset of double targets: HH	HL	LH	LL	MIC	F
BJ	300	305	328	339	288	293	296	322	21	6.2*
RS	297	334	334	518	270	282	282	356	62	54.8***
JS	396	421	439	530	365	384	401	435	15	1.2
MB	286	311	321	484	266	270	277	331	51	28.6***
RM	356	371	394	507	336	344	352	394	34	9.0**
LB	425	479	494	652	371	404	411	515	71	33.8***
JG	240	263	275	433	215	222	226	295	63	147.0***
WY	296	344	333	492	259	280	274	370	75	62.8***
AW	230	247	262	462	216	217	211	274	62	112.0***
Means	314	342	353	491	287	300	303	366	50	

* p<.05; ** p<.01; *** p<.001

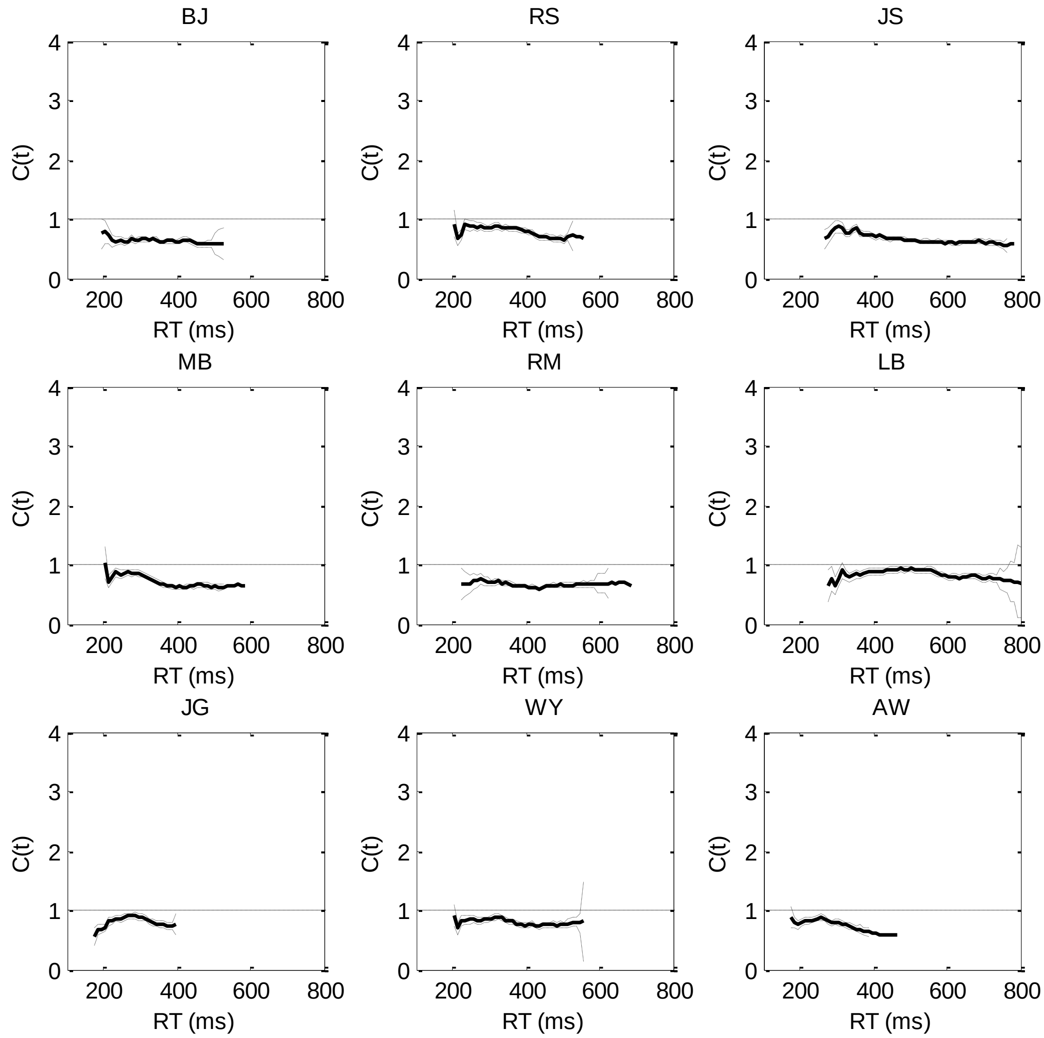

Figure 2. Capacity coefficient values for individual observers in Experiment 1 (OR task). The thin dashed lines represent ±1 standard error of the estimate of the capacity coefficient function (estimated by bootstrapping).

Focusing on the subset of double target trials, HH trials were processed faster, on average (287 ms), than HL (300 ms) and LH (303 ms) trials. LL trials were the slowest (366 ms). Statistically significant main effects are necessary to draw architectural inferences from the interactions. A 2x2 ANOVA for top-target salience (high, low) by bottom-target salience (high, low) revealed significant main effects at $p<.001$ for both factors and for all participants:

Responses were faster when the top target was highly salient (293.5 ms when averaged across all participants) compared to trials where the top target had low salience (334.5 ms). Similarly, responses were faster when the bottom target was highly salient (295 ms across all participants) compared to trials where the salience of the bottom target was low (333 ms). This information also supports the validity of selective influence of the salience factor.

The most pertinent test for the purpose of models' diagnosis is the interaction of salience manipulations of the top and bottom targets, which is in fact comprised of the mean interaction contrast (MIC) and the survivor interaction contrast (SIC) analyses. The analysis revealed significant interactions for all participants but one (JS), ruling out serial models as a viable explanation for the processing of the top and bottom targets. MIC values and the corresponding F values are presented on the two right-most columns of Table 3. MIC values were positive for all participants, supporting parallel processing with a minimum-time stopping rule. Applying the interaction contrast at the distributions' level resulted in SIC functions that were positive for all participants, for all time t (except for JS, for some t), further bolstering a parallel minimum-time mode of processing (Figure 3).

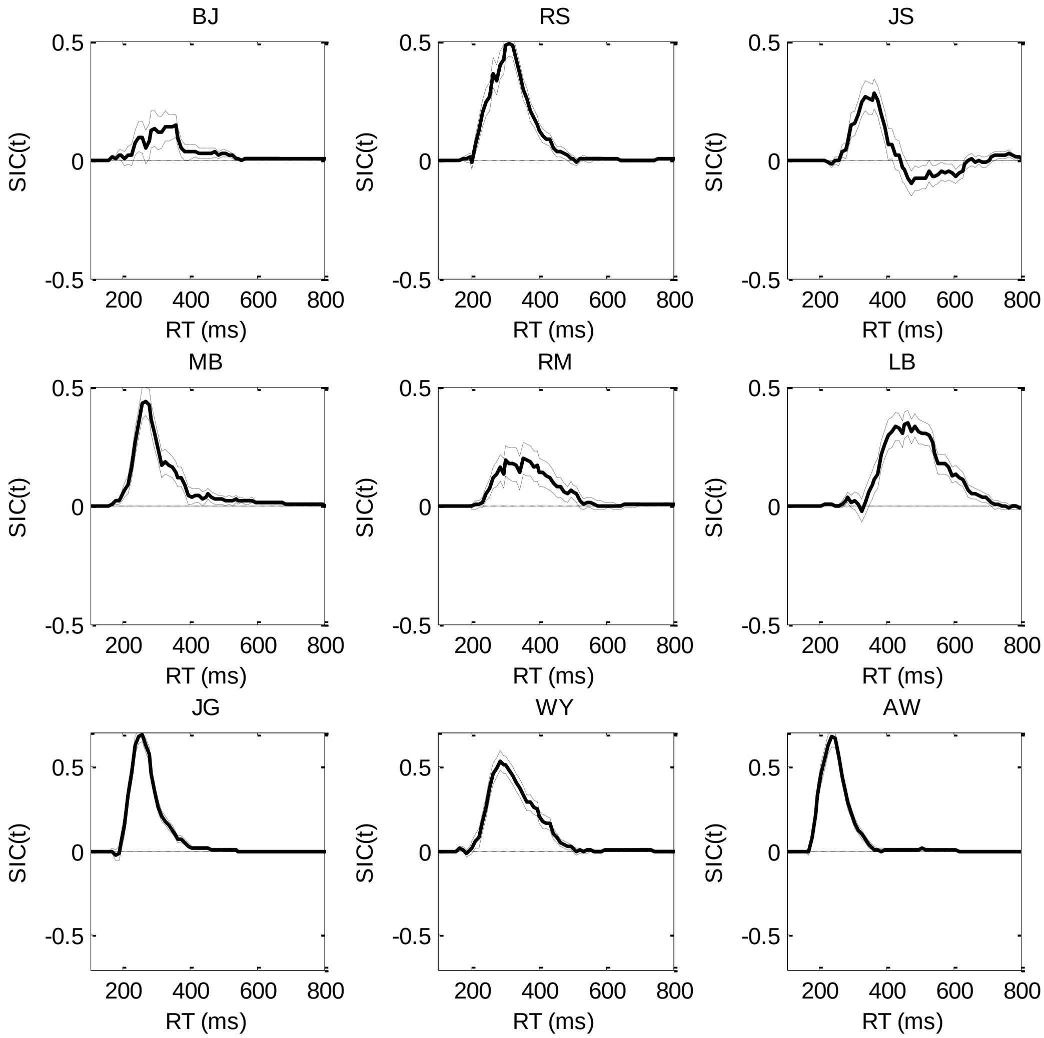

Figure 3. Survivor interaction contrast functions for individual observers in Experiment 1 (OR task). The thin dashed lines represent ±1 standard error of the estimate (estimated by bootstrapping). Scaling of the y-axis may slightly vary across individual plots.

Experiment 2 (AND): Mean reaction times for individual participants are presented in Table 4. For data pooled across participants, mean response time on the double target condition was the slowest (454 ms). This was also true at the individual level for six out of the nine participants. The exact order of the remaining factorial conditions (single-target and no-target) varied across participants.

We performed the same analysis of variance which we have used for the OR data on the response time data from the AND experiment, with the factors top-target (present, absent) by bottom-target (present, absent). The ANOVA revealed significant main effects at $p<.01$ for both factors for seven out of nine participants. These effects, interestingly, were opposite to those observed in the OR experiment: participants were *slower* to respond when a target was presented on the top position (439 ms, averaged across all participants) compared to trials with no target on top (411.5 ms). Concomitantly, they were slower to respond when a target was presented at the bottom position (431.5 ms across all participants) compared to trials with only blank at the bottom (419 ms). One participant (RS) exhibited a significant main effect for the presence vs. absence of the top target [$F(1, 1) = 13.85$, $p<.001$] but not for the bottom target [$F(1, 1)=.35$, $p=.55$]. Another participant (MB) exhibited the opposite pattern [$F(1, 1)=.23$, $p=.63$, and $F(1, 1)=9.41$, $p<.01$ for the top and bottom targets, respectively]. Eight participants (all except LB) exhibited significant interactions of top-target presence x bottom-target presence, at $p<.05$.

A word is in order concerning strategies of computing workload capacity in AND designs. Calculations of $C_{AND}(t)$ are complicated by the fact that unlike the OR design, single target trials in the AND case require a “NO” response rather than a “YES” response. This fact means that, since negative decision times are well known to typically be longer than affirmative times, for a number of reasons (see, e.g., Clark & Chase, 1972), capacity could be artificially computed to be higher than it would if a homogeneous response (in these studies, “YES”) were employed in both the single target as well as the double target trials. Although not a perfect solution to the problem, our stratagem is to transfer the single target data (only) from the OR experiment since these require, as do the double target AND trials, a “YES” response. Of course, this technique assumes that the single target response time distributions will be invariant from the OR to the AND blocks. However, the risk is lowered by the fact that the same participants engaged in both the OR and the AND experiments.

Capacity coefficients were therefore computed for each individual by combining the single target data from the OR experiment and then the double target data, from the same individual, from the AND experiment. Individual capacity plots are presented in Figure 4. Again aided by the standard error of estimation, $C_{AND}(t)$ was found to escape from the zone of limited capacity for participants BJ, JS, and LB, for some time intervals. Capacity was overwhelmingly limited for participants RS, MB, RM, JG, WY, and AW. Interestingly, in the case of AND paradigms the presence of base time may actually lead to increased, overestimated $C_{AND}(t)$, as opposed to the underestimation in the OR case (Townsend & Eidels, 2011). However, we note again that base time is not expected to be a major factor in the estimation of C(t).

Table 4. *Mean response times (in ms.) in Experiment 2 (AND task).*

Participant	Double target	Single target top	Single target bottom	No target	Subset of double targets: HH	HL	LH	LL	MIC	F
BJ	353	355	317	328	326	359	371	358	-46	37.1***
RS	467	442	430	451	416	470	489	500	-43	10.1**
JS	492	438	417	348	431	523	507	515	-83	46.3***
MB	398	379	365	408	358	413	413	417	-51	14.9***
RM	505	408	392	432	467	515	520	523	-44	13.6***
LB	556	598	604	644	481	594	574	580	-107	82.1***
JG	396	350	329	333	342	423	414	416	-78	54.6***

WY	496	439	431	390	426	532	535	503	-138	129.3***
AW	423	405	399	395	355	473	449	435	-131	153.8***
Means	454	424	409	414	400	478	475	472	-80	

* p<.05; ** p<.01; *** p<.001

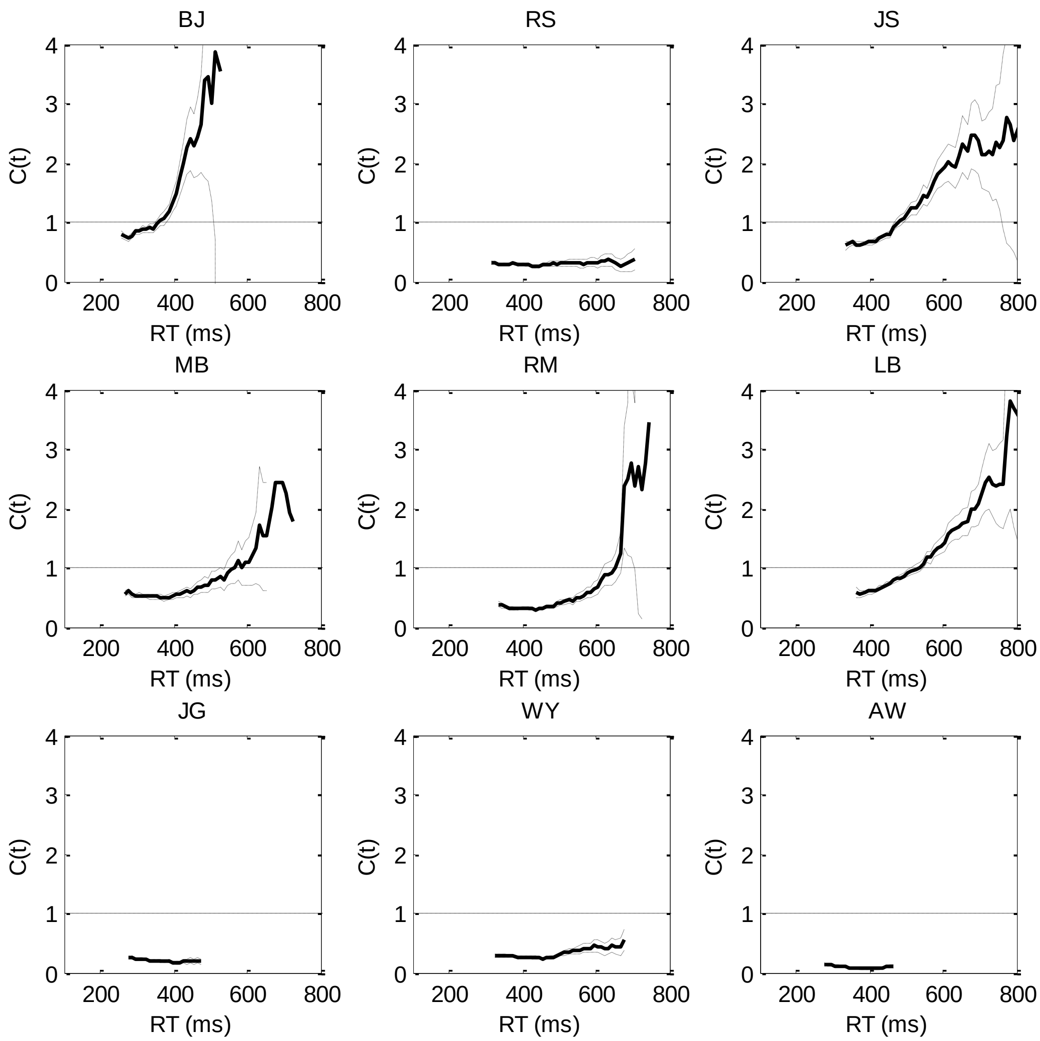

Figure 4. Capacity coefficient values for individual observers in Experiment 2 (AND task). The thin dashed lines represent ±1 standard error of the estimate of the capacity coefficient function (estimated by bootstrapping).

Next, focusing on the subset of double target trials, a close examination of Table 4 reveals that the order of mean response times on the HL, LH, and LL conditions varied from one participant to another. However, mean response times on HH trials were overwhelmingly faster than response times on any of the other luminance conditions, thereby contributing to a negative mean interaction contrast. A 2x2 ANOVA for top-target salience (high, low) by bottom-target salience (high, low) revealed significant main effects at $p<.01$ for both factors, for all participants. As in the OR experiment, responses were faster when the top target was highly salient (439 ms when averaged across all participants) compared to trials where the top target had low salience (473.5 ms). Similarly, responses were faster when the bottom target was highly salient (437.5 ms across all participants) compared to trials where the salience of the bottom target was low (475 ms). Selective influence is again supported.

Recall that the critical test for assessing models' architecture is the interaction of salience manipulations of the top and bottom targets, the mean interaction contrast. MIC values were negative for all participants (at least at $p<.01$; see two right-most columns of Table 4), supporting parallel processing with an exhaustive stopping rule. Applying the interaction contrast at the distributions' level, that is, using the SIC functions resulted in a function that was negative for all time t, further supporting a parallel exhaustive mode of processing (Figure 5).

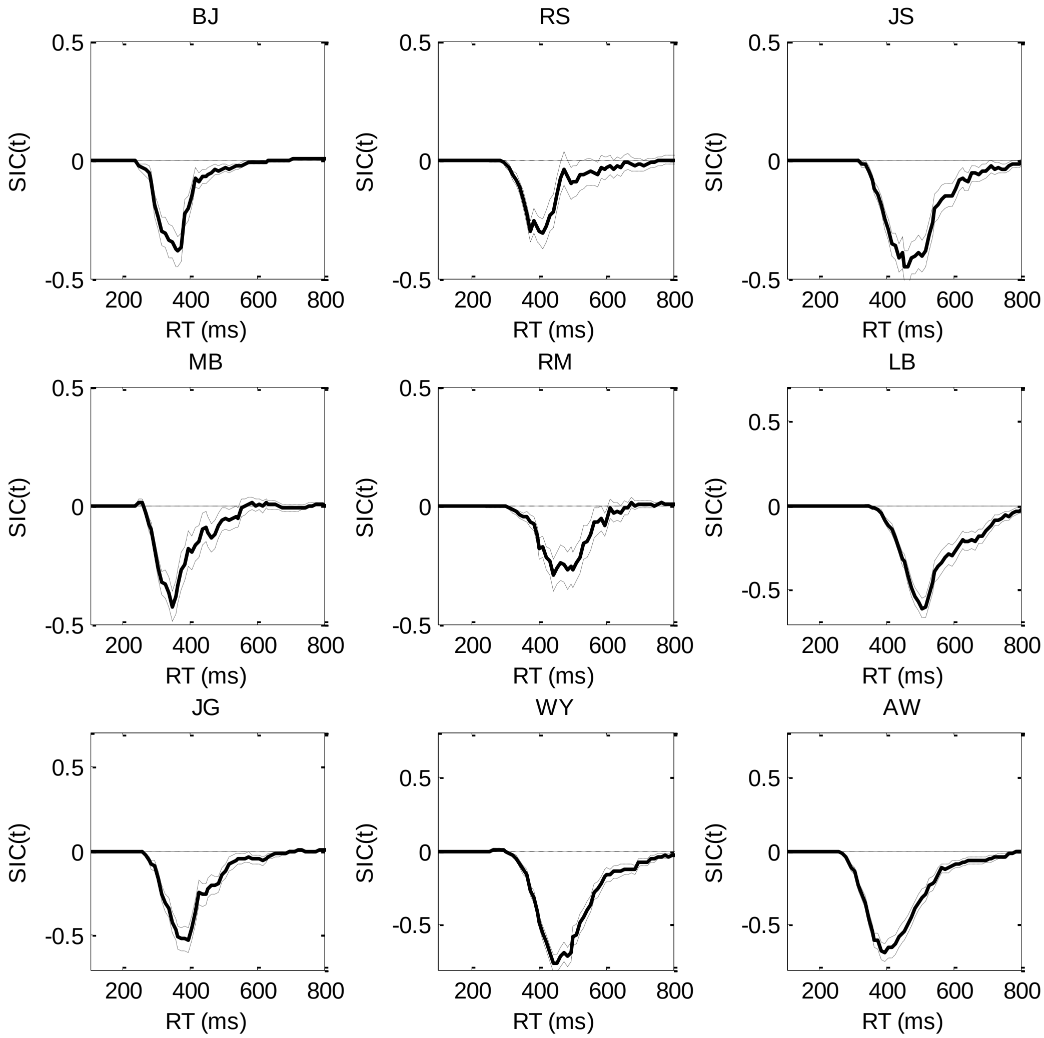

Figure 5. Survivor interaction contrast functions for individual observers in Experiment 2 (AND task). The thin dashed lines represent ±1 standard error of the estimate (estimated by bootstrapping). Scaling of the y-axis may slightly vary across individual plots.

The results of Study I provide valuable information about the architecture and stopping rule of the system when processing simple visual stimuli. The OR experiment, based on a variation of that of Townsend and Nozawa (1995), again finds parallel processing with a minimum-time stopping rule and moderately limited capacity. The new AND experiment also supports parallel processing, but now conforming to an exhaustive stopping rule (both positions must be completed when two targets are present).

Of the Shaw set of models, which seem most like they might be extendable to our RT findings? Overall stochastic parallelism and independence were generally well supported by our RT data, and these are best accounted for by what Mulligan and Shaw (1980) call the Independent-Decision Sharing model. Capacity, however, usually failed to reach the unlimited capacity level, especially in the OR design. In this context, it is interesting to observe a property of the weighted integration model. Namely, though a cousin of our coactive model (which naturally predicts super capacity), this extension of the weighted integration model (when given a time-stochastic interpretation) would have predicted severely limited capacity [$C_{OR}(t) \leq .5$] in OR designs, even worse than the observed data.

We now report Study II, which employs the same response assignments as Study I and almost the same stimuli (except that they are now more difficult to detect) and includes both OR and AND versions. Shaw's model predictions of the NRPC accuracy data are assessed and which of our RT models can handle these results are explored.

Study II: The OR and AND Accuracy Experiments

The same participants from Study I performed in two experimental sessions (OR, AND) on consecutive days. The apparatus was the same as in Experiments 1 and 2, but the targets' luminance was lower in order to make detection more difficult and eventually lead to a substantial proportion of errors. Although some errors are necessary in order to compute the "NO" response probability contrast, Mulligan and Shaw (1980) pointed out that different models are most readily discriminated at high accuracy levels (85-95% correct). Based on a calibration session (method of constant stimuli, 300 trials long -- 20 trials from each of 15 evenly-spaced luminance levels from .001 to .015 cd/m^2, intermixed in a random order) we varied the luminance of targets for each observer such that each individual performed at 85-95% accuracy. Targets' luminance for most participants was set to .005 cd/m^2**.** For two participants (LB, WY) it was set to .012 cd/m^2, for one participant (JS) it was set to .002 cd/m^2, and for another (AW) it was set to .001 cd/m^2.

There were only 4 possible stimulus displays (double target, single target on top, single target at the bottom, and no target; no H or L manipulations), each appearing with a probability of .25. As in Study I, in the OR experiment participants were asked to respond affirmatively if they detected the presence of at least one target, and respond "NO" otherwise. In the AND experiment, participants were asked to respond affirmatively if and only if they detected the presence of two targets, and respond "NO" otherwise. Each experiment started with a 100 trials practice block, followed by 8 blocks of 100 experimental trials each (with 2 minute breaks in between blocks). Participants were instructed to respond as accurately as they can. Due to the task difficulty auditory feedback was provided after each correct (high tone) and incorrect (low tone) response.

Results and Discussion

Overview: The results of Study II, for each individual observer and averaged across all observers, are presented in Table 5 (Experiment 3 -- OR) and Table 6 (Experiment 4 -- AND). For each participant we present the probability of a "NO" response in each of the four factorial conditions (double target, single target on top, single target at the bottom, and no target) and the overall "NO" response probability contrast, NRPC. The reader may find it useful to compare the results with the predictions of the different models in Table 1.

In the OR experiment, NRPC was positive for each of the individual participants, as well as at the means level. In the AND experiment, NRPC was negative for each of the individual participants, as well as at the means level. We further broke down the analysis of each individual participant to 8 blocks, and computed the NRPC separately for each block. In the OR experiment, NRPC was positive on 69 out of 72 blocks (96%). In the AND experiment, NRPC was negative on 66 out of 72 blocks (92%). Appropriate tests for statistical analyses of the NRPC and its transformations are described in Shaw's (1982) Appendix 1. The test for the z score transformation is based on the work of Gourevitz and Galanter (1967).

Table 5. *Probabilities of "NO" responses in Experiment 3 (OR task) for four factorial conditions (no target, single target on top and at the bottom, and targets on both top and bottom positions) and the pertinent "NO" response probability contrast (NRPC) values. NRPC value significantly different than zero is evidence against all-or-none mixture model (and NRPC>0 is a prediction of the sharing model). Log transformation of NRPC that is different than zero is evidence against sharing model (in fact, all independent models). z score transformation of NRPC different than zero is evidence against integration model.*

Participant	P[NO|(T,B)]	P[NO|(T,Ø)]	P[NO|(Ø,B)]	P[NO|(Ø,Ø)]	NRPC	log NRPC	z NRPC
BJ	.01	.17	.03	.93	.75**	.81	2.05**
RS	.03	.31	.07	.97	.63**	.37	2.09**
JS	.01	.13	.03	.97	.81**	.22	2.31**
MB	.01	.44	.03	.91	.46**	-.18	1.11**
RM	.04	.35	.04	.70	.35**	.68	.90**
LB	.04	.20	.04	.91	.71**	1.49**	2.16**
JG	.01	.33	.04	.92	.57**	-.21	1.34**
WY	.02	.07	.06	.97	.86**	1.55**	2.86**
AW	.00	.11	.04	.95	.81**	-1.35	1.62*
Means	.02	.23	.04	.91	.66		

* p<.05; ** p<.01

Table 6. *Probabilities of "NO" responses in Experiment 4 (AND task) for four factorial conditions (no target, single target on top and at the bottom, and targets on both top and bottom positions), and the pertinent "NO" response probability contrast (NRPC) values. NRPC value significantly different than zero is evidence against all-or-none mixture model (and NRPC<0 is a prediction of the sharing model). Log transformation of the YES response contrast (log YRPC) that is different than zero is evidence against sharing model. z score transformation of NRPC that is different than zero is evidence against integration model.*

P's	P[NO|T,B]	P[NO|T,Ø]	P[NO|Ø,B]	P[NO|Ø,Ø]	NRPC	log YRPC	z NRPC
BJ	.24	.98	.97	1.00	-.72**	.71	-1.75**
RS	.42	.97	.98	1.00	-.53**	1.58	-1.56**
JS	.47	.98	.97	1.00	-.48**	.05	-.99
MB	.30	1.00	.96	1.00	-.66**	1.38	-1.81**
RM	.55	.94	.84	.95	-.28**	.75	-.73**
LB	.23	.98	.79	1.00	-.54**	-1.72	-.49
JG	.47	1.00	.94	1.00	-.48**	2.27	-1.67*
WY	.48	.96	.94	.98	-.45**	1.68**	-1.39**
AW	.23	.97	.94	1.00	-.68**	.75	-1.58**
Means	.38	.98	.93	.99	-.53		

* p<.05; ** p<.01

Experiment 3 (OR): A further examination of the OR results in Table 5 reveals that NRPC values were significantly positive for all participants ($p < .01$). These results falsify Shaw's all-or-none mixture model. At the same time, every participant's data are in agreement with Shaw's independent-decision sharing model (NRPC>0). Furthermore, all participants except two exhibit log NRPC values that do not differ significantly from zero and are hence in line with the prediction of the sharing model (NRPC=0). Finally, the z score transformations of NRPC were significantly different from zero, for all of the participants, falsifying the weighted-integration model. This feature will be discussed momentarily.

Experiment 4 (AND): The NRPC values, presented in Table 6, were significantly negative for all participants, in accordance with the prediction of the independent-decision sharing model. We further present in Table 6 the test for logarithmic transformation of the "Yes" response probability contrast, log YRPC = log(P[YES|(Ø,Ø)] – log(P[YES |(A,Ø)]) – log(P[YES |(Ø,B)]) + log(P[YES |(A,B)]). It is trivial to show that for the AND case the independent sharing model predicts log YRPC=0, comparable to the log NRPC=0 prediction of the same model in the OR case (see also Shaw, 1980, p. 378). Only one participant exhibited log YRPC $\neq$ 0, while the other eight exhibited values that did not differ significantly from zero.

Next, the z score transformations of the NRPC were negative for all participants, and significantly different from zero for seven out of nine participants, arguing against the weighted-integration model. For JS and LB, however, these test numbers were not significant. Is it possible that these participants integrated evidence from two channels (corresponding to two spatial locations), as suggested by the model? Or are we facing a statistical power issue? Interestingly enough, in the response time task of Study I, JS and LB also exhibited super capacity [C(t)>1, Figure 4] on the AND, but not on the OR experiment. One explanation is that the AND task encourages integration (at least by some individuals) by virtue of calling attention for one target *and* the other.

In general then, both the response time (Study I) and the NRPC accuracy-based results (Study II) are in agreement with parallel and independent processing of the two signals (except, perhaps, for two participants in the AND task). Further, all the analyses support an appropriate (OR/AND) stopping rule.

General Discussion

Complementary and mutually supportive results were observed for the response time study and the accuracy study. First, we consider the OR and then the AND response time experiments (1 & 2). Then we discuss the accuracy results from Experiments 3 and 4. Finally, the criticality of using both RT as well as accuracy is accentuated by proof that Shaw's time based models can equally well be expressed as a serial or a parallel model.

The RT Study

In the OR experiment of the response time study, participants exhibited a positive (over additive) MIC (mean interaction contrast), from which we tentatively infer a parallel or coactive processing architecture. If the separate decisions assumption (i.e., non-coactive parallel) holds, then a minimum-time stopping rule is indicated.

Next, consider the SIC (survivor interaction contrast) functions: Parallel coactive architectures, where activations from the two channels are integrated (independently, without weighting or interactions in the original channels) into a final common pool, also predict a positive MIC. Recall that coactive models predict a small negative dip in the SIC functions before they go positive (Townsend & Nozawa, 1995; Houpt & Townsend, 2011), but ordinary parallel race models predict continuous positivity. All our OR SIC curves were purely positive disconfirming coactive process models. In addition, coactive models generally predict very high super capacity in the $C_{OR}(t)$ functions. Only extremely high capacity limitations, as when there is massive lateral inhibition (violating the independence assumption), can overcome this tendency (e.g., Eidels et al., 2011; Townsend & Nozawa, 1995; Townsend & Wenger, 2004b). The moderately limited capacity found throughout therefore combines with the SIC functions to render standard coactive (and therefore "integrated" in the standard sense) processing unlikely.

Also, positively interactive first-terminating parallel models tend to produce modest early negative blips and super capacity, like coactive models, but unlike our data (see Eidels et al., 2011). On the other hand, negatively interactive first-terminating parallel models can readily predict qualitatively the same SIC functions as independent parallel systems, but with reduced workload capacity (again, see Eidels et al., 2011). Hence, mild mutual inhibition in a first-terminating system is eminently compatible with our RT results.

The AND experiment delivers data in strong agreement with parallel processing in league with an exhaustive decisional stopping rule. The MIC data were negative as were the individual SIC functions across time. The workload capacity functions were mostly below 1 for small to moderate RTs, but in five of the nine cases, increased to become super capacity for larger time values. We decisively rule out coactivation because: i. Pure coactive processing causes super capacity for all $t > 0$. ii. Coactivation models predict MICs greater than zero and mostly positive SIC functions with modest leading negative blips, even with an AND design, contrary to the results. Of course, a hybrid system where processing starts limited and evolves into a coactive system cannot be ruled out. Another, perhaps more likely possibility is an interactive, separate decisions parallel system with negative interactions to begin with, changing to positive interactions later on. The interactions cannot have been so great as to force deviations from the classical all-negative exhaustive parallel predictions for the SICs, which can occur with extreme interactions (Eidels et al., 2011).

The Accuracy Study

Inferences from the response probability contrast statistics are in striking agreement with the response time conclusions. The "NO" response probability contrast (NRPC) from the accuracy OR experiment is overwhelmingly positive as predicted by the Mulligan and Shaw (1980) independent sharing model, which can be viewed as an extension of our independent parallel model with first-terminating stopping rule.

One might hypothesize that in para-threshold conditions, the system adopts and favors some kind of integration from multiple channels (such as coactivation or weighted integration) to improve detection. However, the results from Study II converge with those from Study I to falsify the weighted integration model as well as the coactive model. Thus, the operative mode of processing, whether stimuli were easy to detect or difficult to perceive, was parallel and close to independent (except for a few interesting exceptions in the AND case). Capacity (from Study I) was either mildly limited, as in the OR design or partly limited, partly super as in the AND case.

Having summarized and interpreted our results, we now turn to a theoretical discussion concerning the static nature of Shaw's models. First, as we expressed before, these models lack temporal specifications. Therefore, we deemed it critical to explore whether the NRPC predictions also hold for RT-choice models that were designed to account not only for the (perceptual) decision but also for its time course. Second, the lack of temporal specification in Shaw's models suggests they are moot to architectural differences. We show below that although they initially appeal for a parallel interpretation, all four models can be equally well viewed as serial processes (and hence subject to 'model mimicry'). Therefore Shaw's models can be interpreted as either parallel or serial models.

Testing NRPC Predictions with Popular Choice-RT Models

We explored examples of two popular classes of choice-RT models -- counting models and random walk models. We show, either analytically (for the former) or via computer simulations (the latter), that they both predict NRPC(OR)>0 and NRPC(AND)<0, much like the static models studied by Mulligan and Shaw (1980).

Counting Models. For readers who would like to see a rather general type of counting model, but one based on discrete counting processes, we provide an Appendix, where we demonstrate with a simple proof that NPRC(OR) > 0 and NRPC(AND) < 0. A simplified description of a counting model, given two sources of information (say, two possible signal positions -- top and bottom), is presented in Figure 6a. There are two parallel and independent processing channels, each accumulates evidence in favor of the presence of a signal in its respective spatial position. To account for NO responses, the model needs to be augmented to the form presented in Figure 6b: for each source of information (top and bottom positions), there also exists a separate channel for “no information”. Each processing channel accumulates counts until a prescribed number is reached and the winner of the race determines the outcome for this position. Poisson counting processes (e.g., Smith & Van Zandt, 2002; Townsend & Ashby, 1983) are natural exemplars of this general set of models.

Random Walk Models. Ratcliff and Smith (2004), in a thorough review, divided sequential sampling models into two major classes: with absolute criterion (the amount of evidence in favor of a particular response must reach a prescribed criterion value), and relative criterion (evidence for one of the response alternatives must exceed the other by some criterion amount). The previously presented counting models form one type of absolute-criterion model. Random walk models (e.g., Laming, 1968; Link & Heath, 1975), as well as diffusion models (a-la Ratcliff, 1978), are members of the relative-criterion models’ class. Processing in random walk models terminates once the amount of accrued evidence reaches a specified bound.

In adopting the random walk framework for our purposes, the decision as to whether a target signal appears on the top position and/or the bottom position is the outcome of separate random walk processes. A schematic depiction of such model is presented in Figure 6c. The overall architecture is somewhat like that of the counting model (Figure 6b), except that the race architectures for the Top and Bottom positions are each replaced by a random walk process. In each of the two random walk processes (top, bottom) evidence is accumulated, in discrete time units, by making a step towards one bound (“YES, target present”, or simply “target”) with probability p, or towards the other bound (“no target”) with probability 1-p (see Fific, Nosofsky, & Townsend, 2008, for a more detailed description of random-walk simulations). A decision for each process is made once it reaches one of the bounds, and a response in the overall system can be made after combining the outcomes in a logical AND/OR gate (depending on the stopping rule and the nature of the task).

Figures 7a (OR case) and 7b (AND case) show NRPC results from Monte-Carlo simulations of a random walk model, with two separate and parallel random walk processes -- one for the top and one for the bottom position. NRPC is overwhelmingly positive for the OR case and negative for the AND case for performance that is better chance (probability correct > .75). Thus, the simulations of the random walk model reinforce our analytic results and extend it to commonly used cases of RT-choice models.

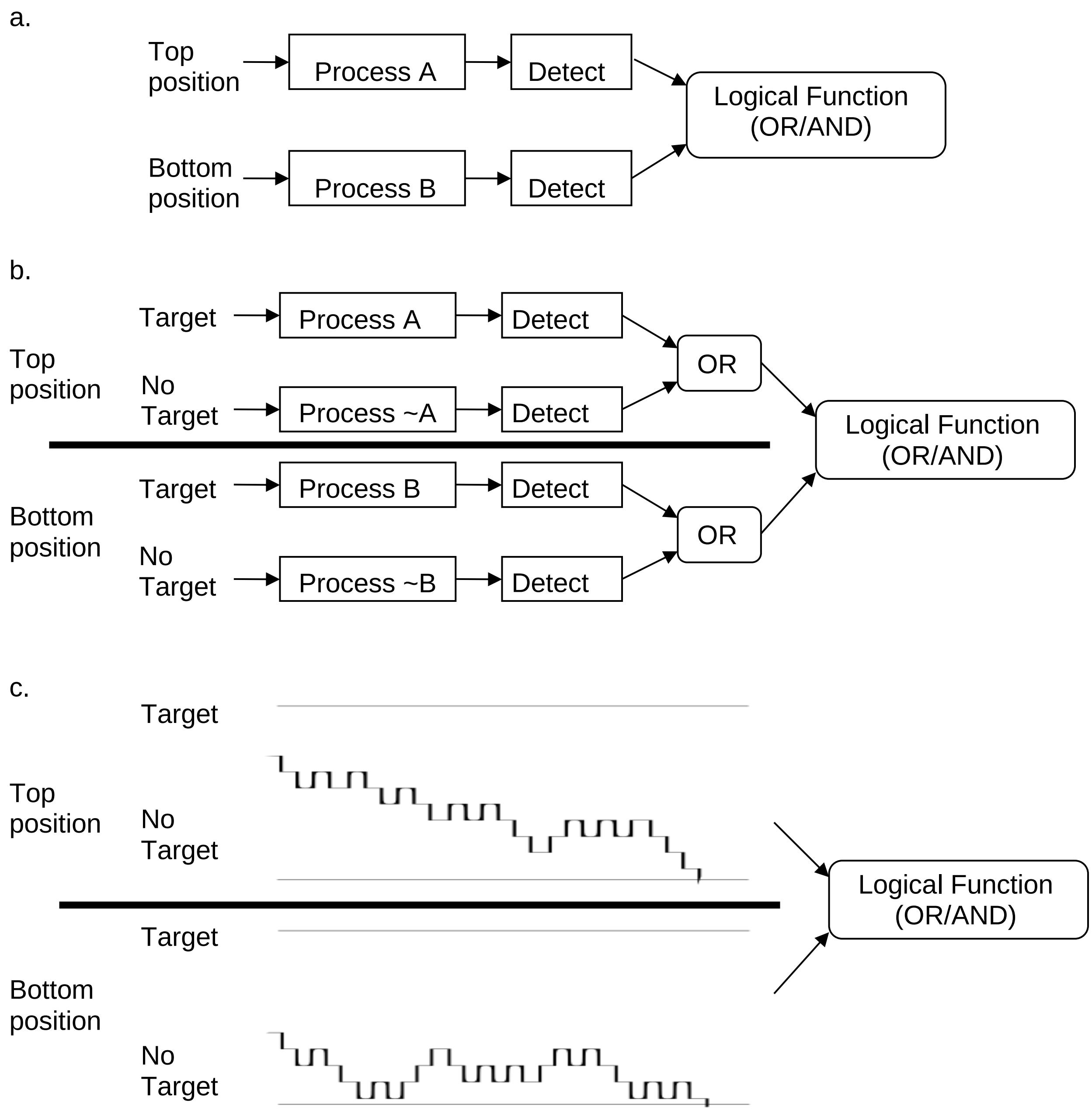

Figure 6. Schematics of a two-channel parallel model (Panel a), a four-channel parallel model (Panel b), and a random-walk model (Panel c). Panel a shows a simplified accumulator model, where evidence from the top and bottom positions is accumulated in separate channels. Panel b illustrates an augmented version, where for each position there also exists a separate “No Target” channel (accumulator). A “NO” response occurs if the “No Target” channel finishes processing before the “Target” channel for that position. The thick solid line indicates that processing of the top and bottom positions is done separately, with separate OR decision gates. Panel c illustrates a parallel arrangements of two random-walk processes, for the top- and bottom targets.

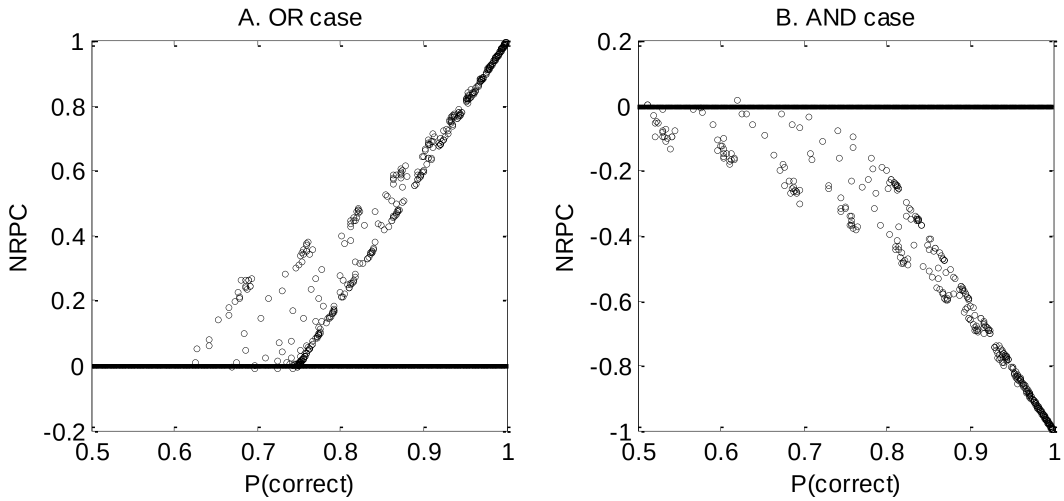

Figure 7. Simulation results of a random walk parallel model (i.e., two simultaneous random-walk processes -- one for the top position, the other for the bottom position). The No Response Probability Contrast, NRPC, is plotted as a function of probability correct for the OR (panel a) and AND (panel b) cases.

We intimated in the previous section that in the absence of temporal specification in Shaw's models, some models can mimic others. The next section is brief, but the messages are important and surprising.

Predictions from Shaw and Colleague's Process Models: Model Mimicking

Although Shaw and colleagues do not place restrictions on the architecture underlying their models, they initially may seem to appeal for a parallel interpretation. We will see that all four models can be equally well viewed as serial processes.

PROPOSITION: *The Mulligan and Shaw (1980) models, due to absence of temporal specification, can all be interpreted as either parallel or serial models.*

Proof:

A. The Independent-Decision Sharing Model. The formula for the likelihood of a NO response (of course conditional on the stimulus compound) is provided at the top of Table 2. i.

The parallel interpretation is naturally that each of two channels measures the activation and a NO occurs if and only if neither channel equals or exceeds its criterion, in a stochastically independent fashion. ii. The serial interpretation is that, independent of order of processing (see, e.g., Townsend & Ashby, 1983), information is acquired on each position (item, etc.) and again, a NO occurs if and only if neither channel equals nor exceeds its criterion, in a stochastically independent fashion.

B. The Independent-Decision All-Or-None Mixture Model (Table 2, second model from the top). i. A possible parallel interpretation is that with probability α the A channel is processed but the B channel is ignored whereas with probability (1- α), the B channel is processed while the A channel is disregarded. ii. The serial account states that with probability α, channel A is operated on first and the decision is made on the evidence from that channel whereas with probability (1- α), channel B is processed and the decision is determined by the result on that channel. Mathematically, accounts (i) and (ii) are identical. In both the serial as well as the parallel account, the performance is expected to be markedly sub-optimal since the evidence from one channel is always ignored (see also Footnote 3).

C. The Independent-Decision Not All-Or-None Mixture Model (Table 2, third model). Unlike (B), this model demands exhaustive processing on each trial, whether serial or parallel. With parallel *or* serial processing, this equation is most naturally viewed as a compound model (Townsend & Ashby, 1983, Chapter 5), where different systems, possibly with distinct parameters or even architectures, are applied from-trial-to-trial. i. The parallel interpretation indicates that with probability α, independent simultaneous processing yields both channels as not reaching their criteria. With probability (1- α) the same type of system, but now with reversed criteria is responsible for a failure on each channel to reach its respective criterion. ii. In the serial account, with probability α the A position is processed first and the B position second with criteria β_A and β_B' respectively. In the serial interpretation, with probability (1- α), the B position is processed first and the A second with reversed criteria β_B and β_A'.

D. The Weighted Integration Model. Here, rather than taking a combination of the probabilities or decisions on each position, based on X_A, and X_B , a weighted combination of the actual information, or activation random variables on each position forms a new random variable $X_A + X_B$, which is then compared with a single criterion. Hence, $P(''NO'') = P([w \cdot X_A + (1-w) \cdot X_B] < \beta)$. i. The parallel rendition is that the actual outputs of each channel are weighted and then added and the decision made. ii. The serial interpretation says that each position is observed one-at-a-time and then the outputs are combined in this weighted fashion before the result is compared with a criterion β. □

Conclusions

Overall, the present set of binocular experiments conjoin the accuracy strategy put forth by Shaw and colleagues (Mulligan & Shaw, 1980; Shaw, 1982) with our response time methodologies to boost support for parallel processing combined with appropriate decision rules in both AND and OR milieus. Although our experiments were quite disparate from those of Mulligan & Shaw (1980), including different retinal locations than theirs (as they called for), our OR accuracy findings were the same as theirs. Serial processing models are decisively falsified by our response time methodology. The overall results also disconfirm standard integration or coactive theories but the super capacity verdict, for some participants in the AND response time experiments calls for further investigation. To the best of our knowledge, there is no parameterized process model which captures all of the present findings, so future efforts may focus on constructing such a system. Bundesen's theory (e.g., 1990, 1993, Bundesen & Habekost, 2008) can handle those results indicating independent, limited capacity parallel processing. However, it fails to predict the shapes of RT distributions that are found empirically (Ratcliff & Smith, 2004).

On the theoretical side, we proved (in the Model Mimicry section of the Discussion) that Mulligan and Shaw's (1980) models, due to the lack of temporal specification, can be interpreted as either parallel or serial models. This mimicry problem of accuracy-based measures and models highlights the value of converging evidence from both RT and accuracy. In conclusion, the conjoining of response times with accuracy appears to auger a promising future for identifying architecture, independence, decisional stopping rule and workload capacity.

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Appendix: "NO" Response Probability Contrast for General Parallel Counting Models

The "NO" Response Probability Contrast, NRPC, with two possible target signals, A and B, can be expressed as P[NO|(Ø,Ø)] – P[NO|(A,Ø)] – P[NO|(Ø,B)] + P[NO|(A,B)].

1. NRPC for the OR case

Since a "NO" response in an OR task is an event in which the number of counts in the channels processing the top position and the bottom position (N_A, N_B) failed to reach criterion (k), we can write the NRPC of a parallel independent model as follows:

$$NRPC_{OR} = P[(N_A < k|\, Ø) \cap (N_B < k|\, Ø)] - P[(N_A < k|\, A) \cap (N_B < k|\, Ø)]$$
$$- P[(N_A < k|\, Ø) \cap (N_B < k|\, B)] + P[(N_A < k|\, A) \cap (N_B < k|\, B)]$$

$$= P(N_A < k|\, Ø) \cdot P(N_B < k|\, Ø) - P(N_A < k|\, A) \cdot P(N_B < k|\, Ø)$$
$$- P(N_A < k|\, Ø) \cdot P(N_B < k|\, B) + P(N_A < k|\, A) \cdot P(N_B < k|\, B)$$

$$= P(N_B < k|\, Ø) \cdot [\, P(N_A < k|\, Ø) - P(N_A < k|\, A)]$$
$$- P(N_B < k|\, B) \cdot [\, P(N_A < k|\, Ø) - P(N_A < k|\, A)]$$

$$= [\, P(N_A < k|\, Ø) - P(N_A < k|\, A)] \cdot [\, P(N_B < k|\, Ø) - P(N_B < k|\, B)]$$

where $P(N_A < k|\, Ø)$ is the probability that the number of counts in the channel processing the top position (N_A) had not reached criterion (k) when no target was presented at this position, and $P(N_A < k|\, A)$ is the probability of not reaching criterion when a target was presented at the top position. $P(N_B < k| Ø)$ and $P(N_B < k| B)$ are the corresponding probabilities for the channel processing the bottom position.

Now, the probability that the number of counts, N_A (N_B), failed to reach criterion when no target was displayed at a particular position has to be larger than the probability of not reaching criterion with a target presented at this position, such that $P(N_A < k| Ø) > P(N_A < k| A)$, and $P(N_B < k| Ø) > P(N_B < k| B)$. Thus, the two square bracketed terms are positive, and so is their product -- the $NRPC_{OR}$. □ Q.E.D.

2. NRPC for the AND case

In an AND task a "NO" response is an event in which the number of counts (N_A and N_B, for the top and bottom positions, respectively) in at least one of the processing channels failed to reach criterion (k), we can write the NRPC of a parallel independent model as follows:

$$NRPC_{AND} = P[(N_A < k| Ø) \cup (N_B < k| Ø)] - P[(N_A < k| A) \cup (N_B < k| Ø)]$$
$$- P[(N_A < k| Ø) \cup (N_B < k| B)] + P[(N_A < k| A) \cup (N_B < k| B)]$$

$$= P(N_A < k| Ø) + P(N_B < k| Ø) - P(N_A < k| Ø) \cdot P(N_B < k| Ø)$$
$$- [P(N_A < k| A) + P(N_B < k| Ø) - P(N_A < k| A) \cdot P(N_B < k| Ø)]$$
$$- [P(N_A < k| Ø) + P(N_B < k| B) - P(N_A < k| Ø) \cdot P(N_B < k| B)]$$
$$+ [P(N_A < k| A) + P(N_B < k| B) - P(N_A < k| A) \cdot P(N_B < k| B)]$$

$$= P(N_B < k| B) \cdot [P(N_A < k| Ø) - P(N_A < k| A)]$$
$$- P(N_B < k| Ø) \cdot [P(N_A < k| Ø) - P(N_A < k| A)]$$

$= [\ P(N_A < k|\ Ø) - P(N_T < k|\ A)] \cdot [\ P(N_B < k|\ B) - P(N_B < k|\ Ø)]$

where, as in the OR case, $P(N_A < k|\ Ø)$ is the probability that the number of counts in the channel processing the top position (N_A) had not reach criterion (k) when no target was presented at this position, and $P(N_A < k|\ A)$ is the probability of not reaching criterion when a target was presented at the top position. $P(N_B < k|\ Ø)$ and $P(N_B < k|\ B)$ are again the corresponding probabilities for the channel processing the bottom position.

And, just as in the OR case, the probability that the number of counts, N_A (N_B), failed to reach criterion when no target was displayed at a particular position has to be larger than the probability of not reaching criterion with a target presented at this position, such that $P(N_A < k|\ Ø) > P(N_A < k|\ A)$, and $P(N_B < k|\ Ø) > P(N_B < k|\ B)$. Thus, the first square bracketed term is positive whereas the second term is negative. Their product, the $NRPC_{AND}$, must be negative. □ Q.E.D