Perhaps Unidimensional is not Unidimensional

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Miller (1965) identified his famous limit of 7+-2 items based in part on absolute identification – the ability to recognize stimuli which differ on a single dimension, such as lines of different length. An important aspect of this limit is its independence from perceptual effects and its application across all stimulus types. Recent research however, has identified several exceptions. We investigate an explanation for these results which can reconcile them with Miller's work. We find support for the hypothesis that the exceptional stimulus types have more complex psychological representations, which can therefore support better identification. Our investigation uses data sets with thousands of observations for each participant, which allows the application of a new technique for identifying psychological representations: the structural forms algorithm of Kemp and Tenenbaum (2008). This algorithm supports inferences not possible with previous techniques, such as multi-dimensional scaling.

Absolute identification (AI) is the fundamental task of identifying stimuli that vary only on one physical dimension. For example, tone frequency (e.g. Hartman, 1954; Pollack, 1952), tone loudness (e.g. Garner, 1953) or line length (e.g. Lacouture, 1997). In a typical AI task, stimuli are first presented to the participant one at a time, each with a unique label. In the test phase, the participant is then presented with randomly selected stimuli from the set and asked to recall the associated labels.

Miller's (1956) classic paper investigated limits in both short term memory and in AI, and found that 7±2 was not only the number of chunks that can be held in short-term memory, but was also the number of items people could learn to perfectly identify in such a unidimensional stimulus set. The upper limit of Miller's range (nine stimuli) is particularly surprising because it is resistant to many experimental manipulations, including extensive practice (e.g. Weber, Green & Luce, 1977), the number of stimuli in the set (e.g. Garner, 1953) and stimulus spacing (e.g. Braida & Durlach, 1972). Most importantly, this limit appeared to be a fundamental aspect of human information processing rather than a sensory limitation, because the same limit applied to a wide range of stimulus modalities (from electric shocks to saltiness: e.g., Lacouture, Li & Marley, 1998; Pollack, 1952; Garner, 1953)

Despite this longstanding assumption that uni-dimensional stimuli are unable to be learned beyond an upper limit, recent work has identified exceptions. One of Rouder, Morey, Cowan and Pfaltz's (2004) participants was able to learn to perfectly identify 20 line lengths. Dodds, Donkin, Brown & Heathcote (2011) reported related learning effects not only for line lengths, but also for dot separation, line angle, and tone frequency. These findings contradict Miller's theory of a small upper limit to memory processing capacity. This could represent an important finding because a small, or null, effect of learning has been included as a central element of many theoretical accounts of AI and memory (including: Stewart, Brown & Chater's, 2005; Petrov & Anderson's, 2004; Marley & Cook's, 1984; and Brown, Marley, Donkin & Heathcote's, 2008). If Dodds et al.'s

(2011) and Rouder et al.'s (2004) results are taken at face value, they might imply that supposedly fundamental capacity constraints can be altered by practice.

There is, however, an alternative explanation. The number of stimuli that can be reliably identified increases exponentially as the number of dimensions increase (Eriksen & Hake, 1955; Miller, 1956; Rouder, 2001), at least when those dimensions can be perceived independently ("separable" dimensions: Nosofsky & Palmeri, 1996). For example, people are able to identify hundreds of faces, names and letters, all of which vary on multiple dimensions. Or, if an observer could perfectly identify say, seven line lengths and also seven angles, they might be able to identify 49 different stimuli with these *combined* features, such as circle sectors. With an additional assumption, this line of reasoning might reconcile the learning effects observed by Rouder et al. (2004) and Dodds et al. (2011) with the long-standing results of Miller (1956). The extra assumption that is required is that some stimulus sets which vary on just one physical dimension might nevertheless invoke a more complex psychological representation. As with physically multi-dimensional stimuli, more complex psychological representations support richer percepts, perhaps allowing multiple ways to estimate the magnitude of a stimulus and hence better identification.

The stimuli used in AI always vary on just one physical dimension, but this does not guarantee that the corresponding psychological representations are uni-dimensional continua. For example, perceived hue is represented either on a circle or a disc (Shepard, 1962; MacLeod, 2003) and the psychological representation of pitch is a helix (Bachem, 1950) even though the corresponding physical stimuli vary on only one dimension (wavelength, in both cases). In Dodds et al.'s (2011) and Rouder et al.'s (2004) studies, it might have been that those exceptional observers who learned to identify stimuli beyond Miller's limit managed this feat by constructing more complex psychological representations for the unidimensional stimuli. If these observers had access to percepts on dimensions that are even partially independent, this could explain their improved performance without challenging Miller's long-standing hypothesis that performance on any *single* dimension is severely limited.

Examining Psychological Representation

In the absence of additional evidence, there is an unsatisfying circularity to this argument. The only evidence that suggests that these physically unidimensional stimuli have more complex psychological representations, is that those same stimuli can be learned. The only tested prediction from the hypothesised complex representation is that those same stimuli can be learned well. One method of independently probing psychological representation is to use multidimensional scaling (MDS; Cox & Cox, 1993; 2001). MDS determines relationships between objects by examining estimates of the perceived similarity of pairs of the objects. In some cases, such as with colour, MDS techniques are able to reliably infer the complex psychological representation extracted from apparently unidimensional stimuli. This success presumably depends on the clear and consistent form of the representation across different people – allowing data to be averaged across subjects. In turn, the consistency of the psychological representation across subjects is probably an upshot of the basic physiology of the retina. In less clear-cut cases MDS is not always sensitive to subtle or inconsistent changes in the form of psychological representations.

Dodds, Donkin, Brown and Heathcote (2010) collected similarity ratings for line lengths, which was one of the stimulus types that Rouder et al. (2004) and Dodds et al. (2011) identified as an exception to Miller's (1956) limit. Dodds et al. (2010) found that MDS was not reliably able to distinguish between one- and two-dimensional representations. The problem is that it lacks a framework for inference about these arrangements. This means that, if one wishes to recover the number of dimensions that best represent a relationship between objects, the conclusions are based on subjective judgements. Lee (2001) investigated this problem in detail and found that, for one- or two-dimensional representations, MDS correctly identified the number of dimensions only 14% of the time.

A recent advance in estimating the structure of psychological representations provides an alternative to MDS. Kemp and Tenenbaum (2008) developed an algorithm that to infer the structure of psychological representations based on relational data. Their method is based on a universal grammar for generating graphs, and the generality of those graphs allows the algorithm to represent structures as varied as trees, hierarchies, and points in vector spaces (as in MDS). An important benefit of Kemp and Tenenbaum's algorithm is that it includes a coherent framework for inference, allowing probabilistic comparison of different structural forms based on penalized likelihood, where the penalty term depends on structural complexity.

We use Kemp and Tenenbaum’s (2008) algorithm to investigate the psychological representation of the stimuli used in AI experiments. We limited our search to undirected graph structures only, on the assumption that the similarity of two stimuli should not depend on the order of comparison (or, if it did, that this dependence was not of primary interest). We also limited our search to just two of Kemp and Tenenbaum's forms – the chain and ring (see Figure 1). Chain structures are the standard assumptions for AI stimuli: one-dimensional continua, where the psychological distance between stimuli is found by summing the distance from one neighbour to the next, and the next again, and so on. Ring structures represent just a small increase in complexity from chains, capturing the additional property that stimuli near one end of the set might be perceived to have something in common with stimuli at the extreme other end. This kind of relationship is found in both of the well-known cases of physically unidimensional stimuli having multidimensional psychological representations: long wavelength light has a perceived hue (red) which is similar to the hue perceived for short wavelength light (violet); similarly, the lowest frequency note in an octave (A) is perceived as similar to the highest (G#).

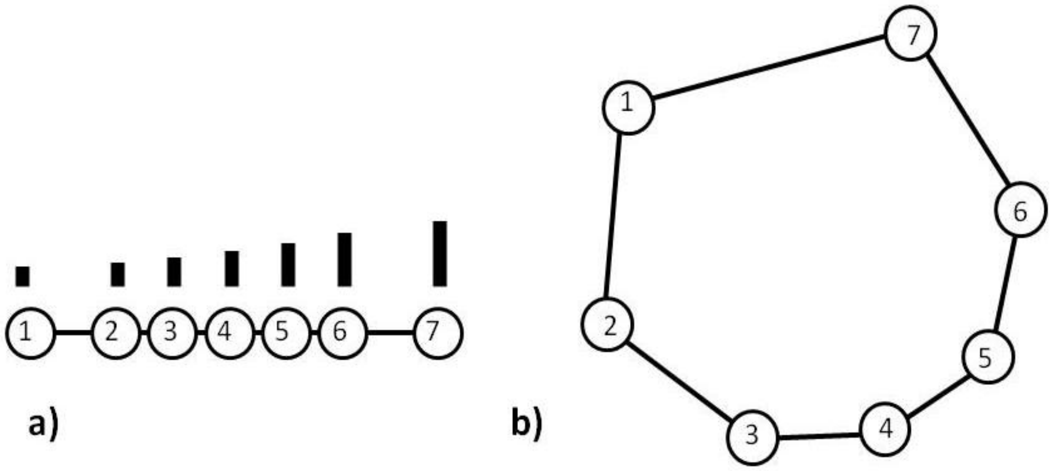

Figure 1. An illustration of a (a) chain structure and a (b) ring structure for lines of varying length. Note that in the chain structure stimuli 1 and 7 are far apart, while in the ring structure they are much closer.

Data

A direct way to investigate psychological structure relies on similarity estimates obtained by direct interrogation: participants are presented with two stimuli and asked to rate their similarity on some scale. Such ratings have many problems. Firstly, there is a severe limit on sample size, because participants find it difficult to give many repetitions of these responses. Secondly, the numerical similarity ratings provided by participants depend on the experimenter's choices. For example, different ratings would be provided if the observers are asked to rate similarity from 1-10 or from 0-100, or on a Likert scale, and the precise nature of this dependence is unclear. Even more troubling, it is unclear whether similarity ratings obtained by this method are based on the particular psychological representation of interest: the one underlying AI performance. To circumvent all three problems, we replace similarity judgments with confusion matrices calculated from many thousands of AI trials. These confusion matrices encode how often each pair of stimuli are confused with each other (e.g., when stimulus A is presented, what is the probability that it is identified as stimulus B?). Our assumption is thus that the probability of confusing two stimuli is monotonically related to their similarity.

We calculated confusion matrices using the data from four AI experiments reported by Dodds et al. (2011; see Table 1). In all four experiments, participants were given extensive practice over a series of 10 sessions, leading to around 5,000 observations per participant. Each experiment included five or six participants. Three of the experiments used a smaller number of stimuli (15 or 16) allowing for unconfounded comparison between different stimulus types. These three experiments included the only one in which participants did not exceed Miller's (1956) limit of 7±2 stimuli (tone intensity) and two in which they did (line length and dot separation). The other experiment used 30 line lengths. We included this experiment because it showed some of the greatest improvement in performance with practice. We are wary of direct comparison between smaller and larger set size experiments because of the varying statistical reliability of the data sets. The larger set sizes resulted in one quarter as many observations contributing to each element of the confusion matrix – as few as three observations per matrix element. The penalized complexity used in Kemp and Tenenbaum's (2008) algorithm means that noisier data lead to a preference for simpler structures – a bias towards identifying chain structures, in our case. This might extend to our smaller set size experiments, because even with 5,000 observations in the smaller set sizes, the average number of observations contributing to each confusion matrix element was between 16 and 20. We return to this point in the Discussion.

We also analysed data from a new AI experiment using tone frequency. For this experiment, we gave six musically- trained participants practice with a set of 36 pure sine tones varying in frequency – their frequencies matched the fundamental frequency of the standard piano notes from A3 to G#6. The procedure for this experiment was similar to the procedure outlined for Experiment 6 in Dodds et al. (2011), except that responses were labelled not only with a number (1-36) but also the corresponding piano note name. There were 10 learning sessions providing 4860 identifications per participant.

Results

Table 1. Data sets used from Dodds et al. (2011)

Experiment*	Stimuli	Set Size
1a	Line Length	30
2b	Dot Separation	15
5a	Line Length	16
5b	Tone Intensity	16

* Note: Experiment refers to the experiment number as listed in Dodds et al. (2011)

Confusion matrices were constructed for individual participants, for a) their entire 10 hours of practice, b) the first 5 sessions of practice and c) the last 5 sessions. Data for individual participants were used as opposed to averaged data because of the small number of participants and large individual variation (see Table 2 for variation in accuracy). As described by Kemp and Tenenbaum (2008), feature data were simulated from the confusion matrices.

We used v1.0 (July 2008) of the Matlab implementation of the structural forms algorithm (obtained from the first author's website). For each confusion matrix, we identified the best chain and best ring structure, and recorded their penalized likelihoods. In all cases, we used default values for the algorithm's parameters. We made one modification to the algorithm, for numerical stability, restricting the search over edge lengths to disallow lengths that were extremely close to zero (smaller than e^{-10}). Note that this restriction still permits edge lengths of precisely zero, because adjacent stimuli can be collapsed into single nodes using the rules of the graph grammar. Our restriction only disallows extremely small, but non-zero, separation between stimuli. In Table 2, we report differences in penalized log-likelihood between the chain and ring structure fits. To put the likelihood results in statistical perspective, differences in log-likelihood can be used to approximate the posterior probability that one model out of the pair (chain or ring) was the data generating model. This approximation should be interpreted with some care, as it relies upon some strong assumptions - for example, that the data generating model was one of the pair under consideration (e.g., Raftery, 1995). Nevertheless, using this interpretation, a difference in log-likelihood of two units corresponds to about three-to-one odds in favour of one model over the other, and a difference of six units in log-likelihood to better than twenty-to-one odds.

Whole Data Sets

We report the analyses for smaller set sizes (15 or 16 stimuli) separately from the larger set sizes (30 or 36 stimuli). This allows cleaner comparison within each group because the number of data per entry in the confusion matrices are comparable: about 18 observations per entry for the small set sizes, and about 4 for the large set sizes.

Small Set Sizes

Small set size experiments included those that used 16 tones of increasing loudness, 16 line lengths and 15 dots varying in separation. Participants who practiced tone loudness did not improve their performance much with practice, and their confusion matrices were also unanimously better described by chain structures than ring structures (see Table 2, where positive log-likelihood differences imply support for chain structures over ring structures). These results are consistent with Miller's (1956) original hypothesis that AI is subject to a severe capacity limit when the stimuli really are unidimensional.

In comparison, the confusion matrices for some of the participants who practiced 16 line lengths and 15 dot separations exhibited were better described by the ring structure than the chain. In these two experiments, the ring structure was deemed more likely for only about half of the participants (5 of 11; see Table 2). The support for a ring structure is even more surprising when considering the data used in these experiments: our use of confusion matrices rather than similarity ratings. For example, if asked for a similarity rating, a participant might rate the extreme edge stimuli as very similar, but they still might be very unlikely to confuse those stimuli in an identification experiment. This presumably biases our results towards the chain structure, and yet several participants were still better described by ring structures.

Those five participants for whom the ring provided a better description in these experiments also demonstrated higher initial identification performance, and more improvement with practice. At the beginning of practice (first session), their mean accuracy was 54%, compared with 44% for the participants better described by chain structures, and over the course of practice, those subjects identified as having ring-like representations improved their identification performance by 32% compared with 29% for the chain-like participants. We are hesitant to calculate inferential tests on these differences due to the very small number of participants (five in one group, six in the other).

Large Set Sizes

Table 2 shows accuracy and log-likelihood differences for experiments with 30 line lengths and 36 tone frequencies. Four of the twelve participants demonstrated greater likelihood for a ring structure compared to the chain structure. As with the smaller set size experiments, those who demonstrated a ring structure demonstrated greater improvement in performance (M_{ring} = 0.36) compared to those that demonstrated a chain structure (M_{chain} = 0.22) and also greater pre-practice performance (M_{ring} = 0.36, M_{chain} = 0.22).

Table 2. Accuracy and log-likelihood values for each participant in each of the five experiments.

Experiment (Stimuli)	Participant	Initial Accuracy	Improvement in Accuracy	Overall Log-likelihood Difference *	Early Log-likelihood Difference *	Late Log-likelihood Difference *
Tone Loudness (16)	1	0.34	0.1	8.357	19.326	12.25
	2	0.3	0.19	20.092	18.705	25.209
	3	0.33	0.12	12.508	26.442	17.106
	4	0.27	0.09	24.007	27.554	20.384
	5	0.34	0.08	26.205	23.19	22.165
	6	0.31	0.13	19.078	19.527	17.521
Line Lengths (16)	1	0.58	0.22	-2.542	0.12	-5.116
	2	0.51	0.38	-3.423	0.171	-4.711
	3	0.41	0.31	7.041	12.733	4.816
	4	0.4	0.28	3.344	9.77	-0.223
	5	0.46	0.30	5.143	13.69	-2.046
	6	0.57	0.24	-3.365	-3.874	-4.798
Dot Separation (15)	1	0.44	0.2	7.852	10.378	8.621
	2	0.51	0.37	-0.877	4.088	-5.481
	3	0.53	0.27	4.472	7.229	2.468
	4	0.39	0.37	5.703	11.38	-12.56
	5	0.53	0.41	-2.658	2.36	-1.536
Line Length (30)	1	0.21	0.26	24.556		
	2	0.18	0.1	13.405		
	3	0.29	0.47	-1.426		
	4	0.2	0.1	24.292		
	5	0.31	0.41	57.852		
	6	0.17	0.24	-22.265		
Tone Frequency (36)	1	0.4	0.29	-4.273		
	2	0.59	0.31	-0.394		
	3	0.2	0.13	27.8		
	4	0.21	0.09	24.934		
	5	0.19	0.08	22.549		
	6	0.22	0.14	0.634		

* Note that difference values are calculated by subtracting the likelihood values for ring structures from the likelihood values for chain structures.

Effect of Practice

Dodds et al. (2011) noted that participants improved their performance markedly given practice at AI for all stimulus sets except for tones varying in intensity. In order to examine whether the improvement in performance was associated with a change in psychological structure, we also examined the confusion matrices for each participant in the small set size experiments separately for early (1:5) and late (6:10) practice sessions (See Table 2). We did not examine this split in the data from the large set size experiments because the sample size was too small – an average of fewer than two observations per entry.

For those who practiced tone loudness (Table 2) there was no difference in the estimated structure between early and late sessions: the data from every participant, for both early and late sessions, were always better described by chain structures than rings. For those who practiced line length or dot separation (Table 2), the chain structure was also dominant for early sessions (10 out of 11 participants). For six participants however, the most likely structure changed from a chain to a ring from early to late sessions. Three participants demonstrated a chain structure both in the early sessions and in the late sessions, and one other demonstrated a ring structure in both early and late sessions. No participant demonstrated the reverse switch – from ring to chain structure. Consistent with the hypothesis that high performance in AI is only possible through more complex psychological representations, the single participant who demonstrated a ring structure during early practice also had very high performance in early practice, and the three participants who demonstrated a chain structure even late in practice were amongst the poorest performers late in practice.

A repeated theme in the above findings is that more complex (ring) structures are associated with better identification and with more improvement with practice. To investigate this more formally, we calculated the correlation between both improvement in performance and initial accuracy, and log-likelihood. Both improvement in accuracy and initial accuracy demonstrated a strong negative relationship with log-likelihood difference values, where smaller log likelihood differences (representing a preference for a ring-structure) was associated with greater overall improvement in accuracy (*r* = -.70, *p* < .001) and greater initial performance (*r* = .65, *p* < .001; see Figure 2).

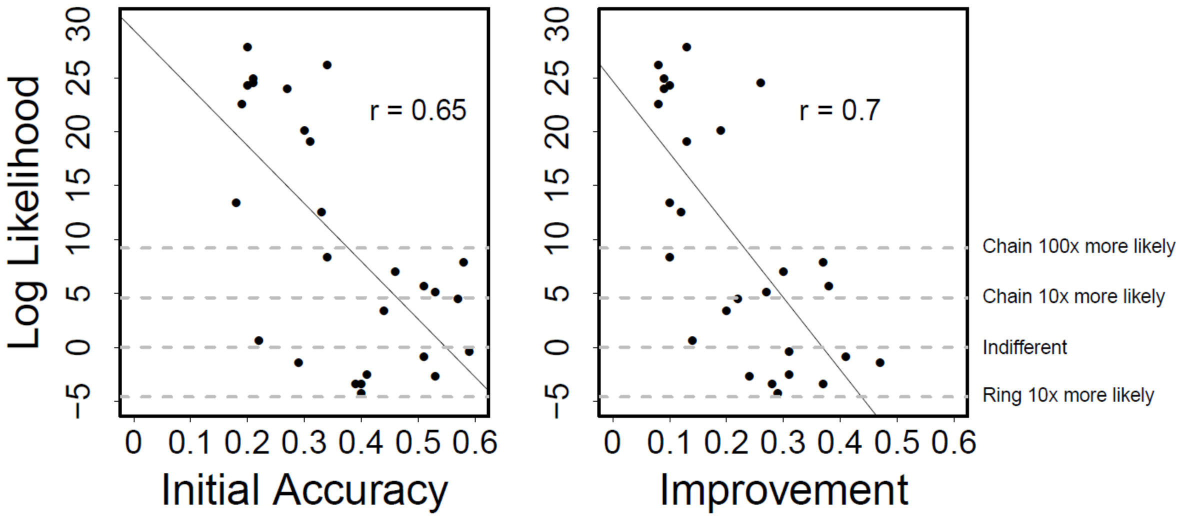

Figure 2. Accuracy and Improvement in accuracy as a function of difference in Log-Likelihood values for ring and chain structures (where a negative Log-Likelihood value indicates a preference for a ring structure). Note: two outliers were removed from this analysis (where log-likelihood difference was < -10 and >30).

Discussion

For more than fifty years, AI with unidimensional stimulus sets has been assumed to be subject to a strict performance limit, Miller's (1956) magical number 7+-2. More recently, Rouder et al. (2004) and Dodds et al. (2011) have shown that some stimulus sets support much greater performance than this limit (including line length or angle, and tone frequency) while at least one does not (tone intensity). One way to reconcile these new findings with previous literature is to hypothesize that some stimulus sets, while physically varying on only one dimension, give rise to a more complex psychological representation. The data from

Dodds et al., and the new structural forms algorithm developed by Kemp and Tenenbaum (2008) provide a method for investigating this hypothesis in a way that was not previously possible because of limitations in analytic tools such as multidimensional scaling.

Our results provide consistent support for the previously untestable hypothesis that improved identification performance is only possible with more complex psychological representations. When we examined data from the identification of tones varying in intensity (for which identification performance was severely limited) we uniformly found strong support for the simplest unidimensional psychological representation – a chain, as assumed in all theoretical accounts of identification. This result was observed for all participants, and was also confirmed as the most likely structure in both the early and late practice data. Data from those stimulus sets for which Dodds et al. (2011) found significantly improved performance with practice yielded different results. The psychological representations of the stimuli for the more than one third of these participants (9 of 23) were better described by the ring structure than the chain structure. This figure rose to 8 of 11 participants when only data from the second half of practice were considered, in stark contrast to the 1 of 11 participants identified as using a ring structure in the first half of practice. The hypothesized relationship between identification performance and structure was further supported by strong correlations between performance in practice and the goodness-of-fit of the ring and chain structures.

To check our results with data from another laboratory, we also analysed data from two of Rouder et al.'s (2004) participants. Those participants practiced line length stimuli in a similar procedure to that described above, using set sizes of 13 line lengths in one experiment and then 20 line lengths in another[1]. Both participants in the first experiment, and one out of two of the participants in the second experiment were better described by the ring structure.

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[1] We did not analyse data from Rouder et al.'s 30-length experiment, as the sample sizes became prohibitively small.

The participants that demonstrated a more complex structure were also those that demonstrated higher initial accuracy (M_{ring} = .85; M_{chain} = .68).

A natural question arising from our analyses is why we did not observe uniform results. That is, if improved performance in the identification task really is supported by more complex psychological representations of the stimuli, why did we not observe such representations for *every* participant? Two explanations seem plausible. First of all, in all experiments there was considerable variability amongst the participants in identification performance. About half of the participants did not learn to improve their performance beyond Miller's (1956) limit of 7±2 stimuli, and so it is consistent with the hypothesis that those participants should maintain the simplest (chain) psychological representations. Secondly, there is an inherent bias favouring the chain structure over the ring structure in noisy data. This bias arises because general noise (such as non-task-related responses, and random error) bias the confusion matrices towards uniformity, and uniform confusion matrices are – according to the structural forms algorithm – better described by chain than ring structures due to the higher complexity penalty attracted by ring structures.

Our results indicate that better performance through practice in identification is associated with more complex psychological representations of stimuli. However, the results do not provide insight into exactly how those representations arise, nor what extra stimulus information is being represented. For example, it is easy to speculate that participants might learn to judge line lengths using information from several sources – perhaps the extent of the retinal image, or the magnitude of the saccade needed to traverse the line, or even cues gained by comparing the line to external objects such as the display monitor. Magnitude estimates obtained from these sources would presumably be highly, but not perfectly, correlated, which could result in psychological representations more complex than chains. Further study might examine such hypotheses by attempting to limit the information available from such cues, for example by presenting visual stimuli using virtual reality goggles.

In summary, it seems that tone loudness was the only stimulus modality that showed consistent evidence for only a single underlying psychological dimension. Line length, dot separation and tone frequencies showed evidence for more complex psychological representations than simple chain structures - particularly for highly-performing participants and post-practice data. The implications of these results are remarkable for the study of memory in terms of AI – if these stimuli are truly represented on multiple dimensions, unidimensional AI does not apply to these stimuli. In the extreme, it might be that the long history of study of unidimensional AI should have been limited to the study of tones varying in loudness. Or in the very least, that the identification of other stimulus types only qualifies as unidimensional as long as participants are not well practiced.

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References

Bachem, A. (1950). Tone height and tone chroma as two different pitch qualities. *Acta Psychologica, 7,* 80-88.

Braida, L. D., & Durlach, N. I. (1972). Intensity perception: II. Resolution in one-interval paradigms. *Journal of the Acoustical Society of America,51,* 483–502.

Cox, T. F. & Cox, M. A. A. (1994). *Multidimensional Scaling*. London: Chapman and Hall.

Cox, T. F. & Cox, M. A. A. (2001). *Multidimensional Scaling*. London: Chapman and Hall.

Dodds, P., Donkin, D., Brown, S.D., Heathcote, A. (2010). Multidimensional scaling methods for absolute identification data *In S. Ohlsson & R. Catrambone (Eds.), Proceedings of the 32nd Annual Conference of the Cognitive Science Society. Portland, OR: Cognitive Science Society.*

Dodds, P., Donkin, C., Brown, S. D. & Heathcote, A. (2011) Increasing Capacity: Practice Effects in Absolute Identification *Journal of Experimental Psychology: Learning, Memory & Cognition, 37(2), 477-492.*

Eriksen, C. W., & Hake, H. W. (1955). Multidimensional stimulus differences and accuracy of discrimination. *Journal of Experimental Psychology, 50*(3), 153-160.

Garner, W. R. (1953). An information analysis of absolute judgements of loudness. *Journal of Experimental Psychology, 46,* 373-380.

Hartman, E. B. (1954). The influence of practice and pitch-distance between tones on the absolute identification of pitch. *The American Journal of Psychology, 67,* 1-14.

Kemp, C. & Tenenbaum, J. B. (2008). The discovery of structural form. *Proceedings of the National Academy of Sciences, 105,* 10,687-10,692.

Lacouture, Y. (1997). Bow, range, and sequential effects in absolute identification: A response-time analysis. *Psychological Research*, 60, 121-133.

Lacouture, Y., Li, S., & Marley, A. A. J. (1998). The roles of stimulus and response set size in the identification and categorisation of unidimensional stimuli. *Australian Journal of Psychology, 50*(3), 165-174.

Lee, M. D. (2001). Determining the dimensionality of multidimensional scaling representations for cognitive modelling. *Journal of Mathematical Psychology, 45*, 149-166.

MacLeod, D. I. A. (2003). New dimensions in color perception. *Trends in Cognitive Sciences, 7*(3), 97-99.

Miller, G. A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *The Psychological Review, 63 (2),* 81-97.

Nosofsky, R. M., & Palmeri, T. J. (1996). Learning to classify integral-dimension stimuli. Psychonomic Bulletin & Review, 3(2), 222-226.

Pollack, I. (1952). The information of elementary auditory displays. *Journal of Acoustic Society of America, 24*, 745-749.

Rouder, J. N. (2001). Absolute identification with simple and complex stimuli. *Psychological Science, 12*, 318–322.

Rouder, J. N., Morey, R. D., Cowan, N., & Pfaltz, M. (2004). Learning in a unidimensional absolute identification task. *Psychonomic Bulletin & Review, 11 (5)*, 938-944.

Shepard, R. N. (1962). The analysis of proximities: Multidimensional scaling with an unknown distance function. *II Psychometrika, 27 (3),* 219-246.

Weber, D. L., Green, D. M., & Luce, R. D. (1977). Effects of practice and distribution of auditory signals on absolute identification. *Perception and Psychophysics, 22*(3), 223-231.